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EXCITATION AND QUENCHING OF DETONATION IN GASES

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The results of investigations on the problems of initiation, propagation, and stabilization of detonation waves and flowing combustible gaseous mixtures are presented. To describe the flows, we used ideal perfect gas equations and two models of the detonation wave: the classical infinitely thin model and a model in which behind the shock wave chemical reactions described by the single-stage kinetics for propane– and methane–air combustible mixtures proceed. Investigations were carried out by both analytical and numerical methods based on the S. K. Godunov scheme on stationary and movable computational meshes with explicit resolution of the bow shock and the surfaces separating gases with different properties.

Keywords: *detonation, combustion, initiation, optimization, stabilization, channel, structure of the detonation wave, numerical simulation.*

Introduction. The investigation of detonation waves is dictated mainly by the aspiration to use their breaking force for practical purposes in original pulsed plants and special power systems for flying vehicles and rockets. The large values of the gas-dynamic parameters and the complex flow pattern behind the detonation wave front seriously impede both experimental and theoretical studies of this phenomenon.

The main source of information on detonation waves is experiments aimed at revealing details of the process. The quality of the experiment depends to an enormous extent on the personality of the experimenter and on his experience and ingenuity. Among the scientists in the experimental field, R. I. Soloukhin deservedly occupies a special place. His works became handbooks for generations of theorists [1–9].

As scientific notions about detonation widened, the gas-dynamic models taking into account the features of this phenomenon became more complicated. When a complex shock-wave structure of detonation was established in experiments, a demand arose for an adequate gas-dynamic and mathematical model of the detonation wave. The new model proposed by V. P. Korobeinikov and V. A. Levin [10], unlike the popular Zel'dovich–von Neuman–Doering model, was to describe the essentially nonstationary processes behind the detonation wave front. A structure suitable for this purpose turned out to be the system of Euler equations complemented with two model equations of chemical kinetics, one of which describes the ignition delay and the other of which describes the heat release. In the works of different authors, it was shown that this system of equations with initial and boundary conditions describes very well the internal nonstationary wave structure of detonation.

In the first theoretical works, the attenuation laws of one-dimensional, weakly overcompressed, infinitely thin detonation waves for all kinds of flow symmetry were obtained by the analytic method [11, 12]. Later the discovered laws were confirmed by numerical calculations of the flows initiated by a point explosion in a combustible gas [13]. In particular, it was established that a plane detonation wave goes to the Chapman–Jouguet regime at an infinitely remote point, and cylindrical and spherical waves — at an infinite distance from the place of their initiation. From the point of view of the two-stage model, by the small-parameter method the initial stage of flow development at a point explosion was investigated analytically and the effect of splitting of the detonation wave, i.e., a monotonic increase in the distance between the intensive heat-release zone and the bow shock, was revealed [14, 15]. Numerical calculations with the use of model and real kinetics of chemical reactions permitted elucidating the mechanism of initiation and propagation of a self-sustained detonation wave at a concentrated power input. It was shown that such a wave is always nonstationary, and the bow shock parameters change periodically under the action of the shock waves formed in the induction zone before the accelerating flame front [16–20].

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According to the calculations, the self-oscillation process develops only in the case where the explosion energy value exceeds a certain critical value. Otherwise, the detonation wave attenuates, decomposing into a shock wave and a slow-combustion wave. If the explosion energy is close to critical, then detonation attenuation occurs after several oscillations.

In [20–22], critical energy values of detonation initiated by a piston, an electric discharge, an exploding wire, and a trinitrotoluene charge were obtained and their dependence on the combustible mixture parameters and the space-time characteristics of the energy sources was determined. In [21], the anomalous dependence of the critical energy on the electric discharge time known from the experiments of [23] was explained. According to the calculations, this is due to the existence of a characteristic time of energy input during which practically the whole of the gas mass participating in the formation of a powerful shock wave propagating in the combustible mixture flows out of the discharge zone, causing a marked decrease in the density in it. As a result, a large portion of the then input energy is "wasted," i.e., is expended in heating the remaining mass of the gas. The initiation of detonation by a trinitrotoluene charge in an inhomogeneous hydrogen–air mixture was investigated. In particular, the dependence of the critical energy on the parameters of the mixture formed by the diffusion of hydrogen into the air from a point or from a finite spherical volume was determined [24]. It was established that detonation attenuates if the spherical charge is surrounded by an air layer whose outer radius exceeds a certain critical value proportional to the charge radius [25]. For the case where the screening air layer is inside the combustible mixture and does not adjoin the charge, the dependence of its minimal critical thickness on the inner radius and the explosion energy was obtained. On the basis of the calculation data, a formula that permits estimating the possibility of realizing detonation at given energy values and parameters of the air space was proposed [26].

In [27], the possibility of decreasing the critical energy value was explored. It was shown that it decreases by an order of magnitude if the spherical charge is surrounded by a rigid shell of certain radius collapsing in due course after interacting with the bow shock wave. On the basis of the calculations for the cylindrical case and the analogy between nonstationary one-dimensional and stationary hypersonic flows, a method was proposed for stabilizing the detonation wave on a blunt-nosed body around which a hypersonic flow of the combustible mixture takes place with the aid of a rigid shell of certain radius and length surrounding the body [28, 29].

Original mechanisms of detonation initiation were investigated. For instance, the possibility of detonation initiation in a hydrogen–air mixture as a result of the collapse of a low-pressure spherical domain without additional energy input from the outside was established. Calculations performed at various initial radii of the collapsing domain and various pressures inside of it showed that even under normal conditions in the external space upon reflection of the confluent shock wave from the center of symmetry of the flow a self-sustained detonation wave may arise [30–33]. For motions with cylindrical and spherical waves, the dependence of the minimal radius of the domain at which detonation is realized in the environment on the pressure inside of it was obtained.

Within the framework of the investigation of the non-one-dimensional structure of the detonation wave, initially with the use of the two-stage kinetics the process of development of disturbance of the plane detonation wave leading to the formation of the experimentally observable cellular structure of two-dimensional detonation was considered [34]. The existence of minimal and maximal sizes of cells was established and the determining role of transverse waves in the initiation and propagation of non-one-dimensional detonation, in particular, when the wave goes into a divergent channel, was revealed. A two-dimensional model was formulated, by which the structure of the two-head spin detonation wave was calculated [19] and which currently is widely used by many authors to model rotating detonation in applied problems. The wave processes under propagation of detonation in plane channels of complex shape filled with a stoichiometric hydrogen–air mixture under normal conditions with account for the real chemical interaction kinetics were investigated [35–39].

The influence of a nondestructible obstacle (wall) with a height smaller than the channel width located across a channel with parallel walls on the process of propagation of a cellular detonation wave was investigated. It was established that there exists a critical height of the obstacle (depending on the channel width) exceeding which leads to the quenching of the detonation regime of combustion. It was obtained that when the detonation regime of combustion is preserved upon passing over the obstacle, the cellular structure of the detonation wave is recovered only after some time. In the case of quenching of detonation upon interaction with the obstacle, the possibility of its recovery by means of an additional transverse wall placed in the channel was established. In the case where the obstacle whose height exceeds the critical one destroyed with time, the influence of the time of its existence on the process of detonation propagation was investigated. It was established that for the detonation regime of combustion in the channel to be preserved, the existence time of the obstacle should not exceed a certain critical value. The conditions for preserving the cellular detonation wave in a channel with a destructible transverse wall blocking the channel were also determined. The transition of the formed cellular detonation wave from a channel of constant cross section into a sharply diverging channel was investigated. It was established that in the case where the detonation goes into the sharply diverging part out of a channel whose width is smaller than half of the critical width of the channel for the detonation wave to go into open space, the detonation regime of combustion is preserved if the divergence of the channel does not exceed a certain critical value.

The initiation of detonation in a supersonic flow of a hydrogen–air mixture by an electric discharge with spatially homogeneous and inhomogeneous energy release was investigated [40, 41]. The influence of the discharge time and the supersonic flow velocity on the process of detonation formation was studied. The critical energies in initiating detonation by an electric discharge in the form of a flat layer were determined and their dependence on the layer thickness was investigated. It was established that in the considered cases a monotonic decrease in the initiation energy with decreasing layer thickness is observed. It was shown that at relatively large values of the layer thickness the initiation is only due to the portion of the total discharge energy providing the propagation of a bow shock wave of sufficient intensity in a time of the order of the formation time of the zone of stable chemical reactions with intensive heat release. In the case of instantaneous electric discharge with a nonuniform energy distribution across the channel according the sine law, it was established that the critical initiation energy value can be decreased due to the reflection from the channel walls of the powerful transverse shock waves formed upon energy input. At a nonuniform spatial distribution of the electric discharge energy, the influence of the energy input time and the supersonic flow velocity on the process of detonation formation was investigated. In particular, the effect of increase in the initiation energy with increasing discharge time and the flow velocity was established.

The problem of detonation stabilization in the supersonic flow in channels and tubes was also considered. In the two-dimensional formulation, it was shown that a nonstationary upstream propagating detonation wave can be localized due to the energy supply by low-intensity electric discharges at certain instants of time. Within the quasi-one-dimensional approach, the behavior laws of the wave were established and the possibility of its stabilization due to a special shape of the channel was shown.

The rapid development of computer engineering opened up practically unlimited possibilities of modeling various phenomena in nature and engineering with account for their complex fast physicochemical processes. In this work, we present some of the results of theoretical studies connected with the development of prototypes of power plants in which conditions for high-velocity burning of gaseous mixtures of hydrocarbon fuels with an oxidizer are realized.

Mathematical Model and Numerical Method. To describe the flow, we use the system of equations of the gas dynamics of an ideal multicomponent reacting medium:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 , \quad \frac{\partial \rho_i}{\partial t} + \frac{\partial (\rho_i u)}{\partial x} + \frac{\partial (\rho_i v)}{\partial y} = \omega_i ,$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial (p + \rho u^2)}{\partial x} + \frac{\partial (\rho u v)}{\partial y} = 0 , \quad \frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho u v)}{\partial x} + \frac{\partial (p + \rho v^2)}{\partial y} = 0 ,$$

$$\frac{\partial (H - p)}{\partial t} + \frac{\partial (H u)}{\partial x} + \frac{\partial (H v)}{\partial y} = 0 , \quad H = \sum_{i=1}^{N} \rho_i h_i + \rho \frac{u^2 + v^2}{2} .$$

Here ρ_i and h_i are the density and enthalpy of the ith component; ω_i is the rate of change in ρ_i in chemical reactions. The equations of state of the mixture take the form

$$p = \sum_{i=1}^{N} (\rho_i / \mu_i) R_0 T , \quad h_i = c_{0i} + c_{pi} T , \quad i = 1, ..., N ,$$

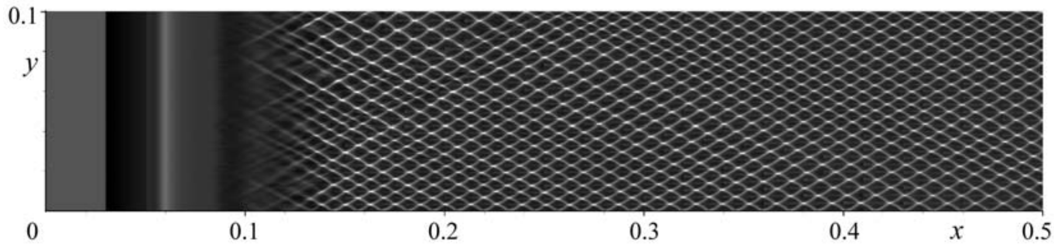


Fig. 1. Traces of triple points in a constant-width channel.

where μ_i stands for the molar masses of the mixture components.

We use the model of single-stage kinetics [42] in which the hydrocarbon fuel combustion is described by one irreversible reaction. Propane–air and methane–oxygen mixtures are used as combustible mixtures. In the case of propane, the reaction proceeds in accordance with the stoichiometric equation

$$C_3H_8 + 5O_2 + 20N_2 \to 4H_2O + 3CO_2 + 20N_2 .$$

Here $N = 5$, and the reaction rate determines all ω_i according to the following equalities:

$$\frac{\omega_{C_3H_8}}{\mu_{C_3H_8}} = \frac{\omega_{O_2}}{5\mu_{O_2}} = -\frac{\omega_{H_2O}}{4\mu_{H_2O}} = -\frac{\omega_{CO_2}}{3\mu_{CO_2}} = AT^{\beta} e^{-\frac{E}{R_0 T}} \left(\frac{\rho_{C_3H_8}}{\mu_{C_3H_8}}\right)^a \left(\frac{\rho_{O_2}}{\mu_{O_2}}\right)^b , \quad \omega_{N_2} = 0 ,$$

where indices i were replaced by the component symbols.

Likewise, for the methane–oxygen mixture $N = 4$, and the reaction is described by the stoichiometric equation

$$CH_4 + 2O_2 \to 2H_2O + CO_2 .$$

The air is assumed to be a mixture of hydrogen with nitrogen in a ratio $\nu_{O_2}:\nu_{N_2} = 1:4$. In so doing, the propane–air mixture is given by the ratio $\nu_{C_3H_8}:\nu_{O_2}:\nu_{N_2} = 1:5:20$.

The investigation was carried out by a modified Godunov method [43] of first order of accuracy in space and time. The method was realized in an original computational complex designed for solving a wide circle of problems of nonstationary dynamics of gaseous combustible mixtures.

Simulation of Two-Dimensional Cellular Detonation in Channels and in Open Space. At the present time, the main qualitative indicator of numerical simulation is the possibility of representing the cellular structure of the flow behind the detonation wave. This section presents the results of calculating flows with the initiation of detonation of the stoichiometric propane–air mixture for three configurations of the flow region: a) in a planar channel of constant width; b) in a channel consisting of a part of constant width and a part with a wedge-like divergence; c) two-dimensional divergent flow behind a cylindrical detonation wave. In cases a and b initiation occurs due to the instantaneous energy supply to the mixture in some zone near the closed end of the channel at which the pressure and temperature increase so that a self-sustained detonation wave is formed quickly. Case c simulates the process of initiation of divergent detonation in the narrow gap between the plates into which the plane detonation wave formed in the tube attached to the plate at a right angle penetrates through a round hole. Such initiation took place in the real experiment [1]. It is assumed that in a circle of small radius with center at the origin of the Cartesian coordinate system there are detonation products with gas-dynamic parameters equal to those behind the self-sustained detonation wave. This zone generates a divergent flow and maintains the initiation process up to a certain instant of time until the divergent shock wave transforms into a detonation wave. Something like that occurs also in the real three-dimensional flow.

Figure 1 shows the calculated pattern of waves of detonation-wave-front bends in case a in a channel of width 0.05 m. One can easily see the formation of a cellular structure which occurs here and in the other cases spontaneously because of the development of weak disturbances introduced due to the rounding-off in the calculations and is associated with the instability of the combustion zone and of the entire process with detonation propagation. Figure 2 demonstrates the transformation of the cellular structure after the detonation wave goes into a wedge-like expansion with

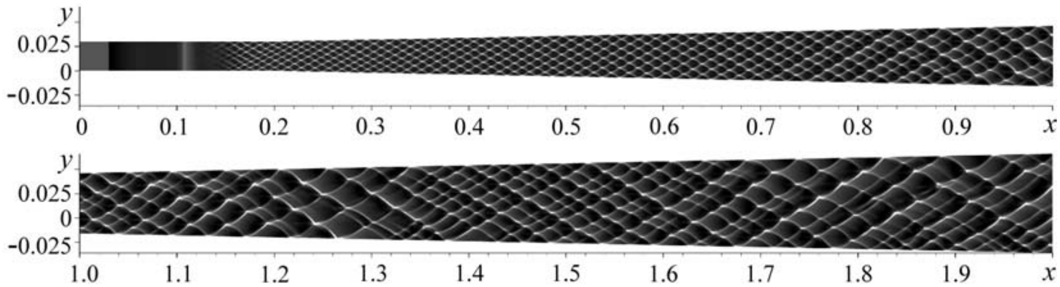
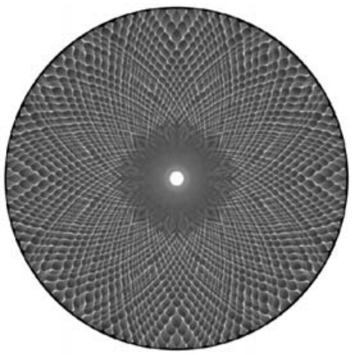


Fig. 2. Traces of triple points in a divergent channel.



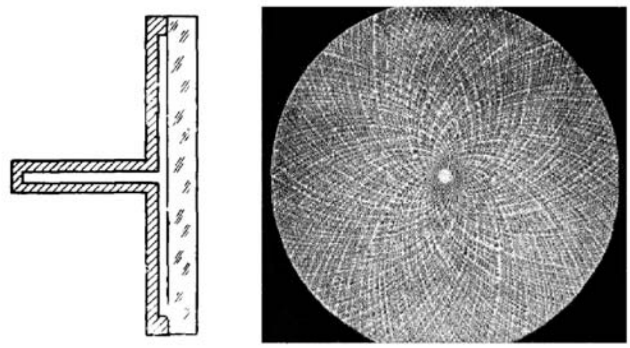


Fig. 3. Traces of triple points behind the diverging cylindrical detonation wave.

Fig. 4. Scheme of the experiment and the cellular structure obtained in [1].

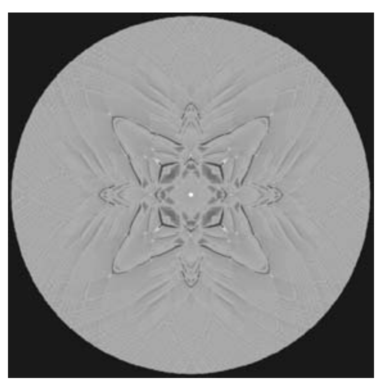


Fig. 5. Traces of triple points behind the diverging cylindrical detonation wave tending to attenuate.

an opening angle of 2.3^{o}. In the given case, the detonation does not attenuate and, as in experiments, this is due to the formation of new cells as the wave propagates. According to the calculations, this occurs in divergent channels with opening angles smaller than a certain critical one. Otherwise, new cells are not generated and the detonation attenuates. Figure 3 shows the pattern of wakes of triple points in case c for the propagation of a divergent cylindrical wave.

Figure 4 shows for comparison the cellular structure obtained in the experiment and the structure taken from [1]. It is seen that there is a good qualitative agreement pointing to both the successful choice of the two-dimensional model of the three-dimensional process and a good quality of the computing experiment. Figure 5 illustrates a situation where, because of the insufficient upthrust, the detonation wave at the initial stage tends to attenuate ,as evidenced by the inhomogeneous wake pattern. Then the detonation goes to the self-sustained regime with a cellular structure.

Optimization of the Thrust Characteristics of the Pulsating Detonation Engine. In this section, we consider the problem simulating the working cycle of the pulsating detonation engine and permitting estimation of its highest possible thrust characteristics. The investigation is carried out from the point of view of the model of an infinitely thin detonation wave analytically and numerically.

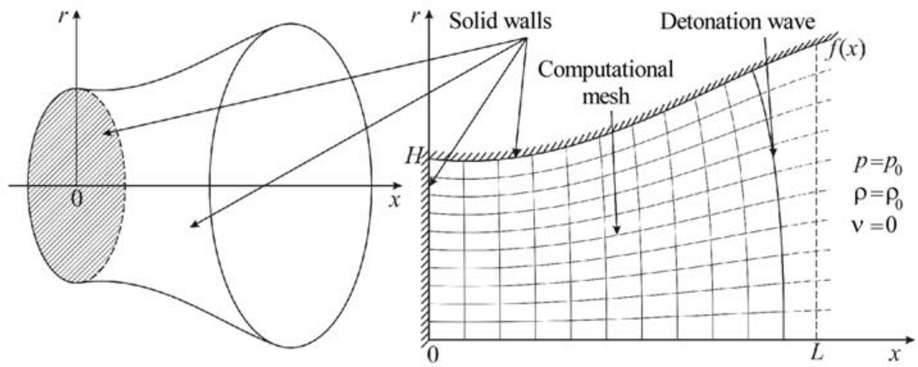


Fig. 6. Illustration of the problem formulation. The detonation wave propagates in the axisymmetric channel from the closed end.

Formulation of the problem. Consider an axially symmetric channel filled with a combustible mixture (Fig. 6). Denote the symmetry axis of the channel by x and the distance to the axis by r. The detonation wave propagating in the mixture causes a flow of detonation products, creating a thrust. Let us assume that the channel has a fixed length L and a flat closed end of radius R, and detonation is initiated along its entire surface. A thrust is created on the closed end of the channel and on its lateral wall. The shape of the lateral wall is given by the function $f(x)$ chosen from a certain class, in particular, from the class of linear or parabolic functions [44].

We consider only the stage of detonation of the working cycle of the pulsating detonation engine which ends as soon as the detonation wave reaches a point on the wall with abscissa L. In such a formulation of the problem, it may be assumed that the channel is infinite, and each intermediate instant of time may be regarded as an end time of detonation with a certain value of the L coordinate depending on the position of the detonation wave front.

To describe the process, we use a model of an infinitely thin detonation wave that permits fairly fast numerical calculations of nonstationary flows, which makes it possible to vary the shape of the lateral wall and obtain quickly the flow parameters for each chosen shape. In order to simplify the formulation of the problem, we assume that the combustible mixture is at rest, is homogeneous, and has constant density $\rho = \rho_0$, pressure $p = p_0$, and temperature $T = T_0$.

The initial mixture and the detonation products are assumed to be ideal perfect gases obeying the caloric and thermal equations of state of the form

$$\varepsilon = \frac{p}{(\gamma - 1)\,p} + \beta Q\,, \quad p = \rho R_{\mathrm{g}} T\,,$$

where $\beta = 1$ in the initial mixture and $\beta = 0$ in the detonation products.

The axisymmetic nonstationary flow behind the detonation wave is described by the system of equations

$$\frac{\partial\,(\rho r)}{\partial t} + \frac{\partial\,(\rho u r)}{\partial x} + \frac{\partial\,(\rho v r)}{\partial r} = 0\,,$$

$$\frac{\partial\,(\rho u r)}{\partial t} + \frac{\partial\,[(p + \rho u^2)\,r]}{\partial x} + \frac{\partial\,(\rho u v r)}{\partial r} = 0\,,$$

$$\frac{\partial\,(\rho v r)}{\partial t} + \frac{\partial\,(\rho u v r)}{\partial x} + \frac{\partial\,[(p + \rho v^2)\,r]}{\partial r} = p\,, \qquad (1)$$

$$\frac{\partial\,(e r)}{\partial t} + \frac{\partial\,[(e + p)\,u r]}{\partial x} + \frac{\partial\,[(e + p)\,v r]}{\partial r} = 0\,,$$

$$e = \rho\left(\varepsilon + \frac{u^2 + v^2}{2}\right).$$

At $t > 0$, on the channel walls the no-flow conditions — equality to zero of the gas velocity component normal to the wall — should be fulfilled, and on the detonation wall the known laws of conservation of mass, momentum, energy, and tangential velocity components should be obeyed:

$$\begin{gathered}
[\rho]\, D - [\rho v_{\mathrm{n}}] = 0\,, \\
[\rho v_{\mathrm{n}}]\, D - [p + \rho v_{\mathrm{n}}^2] = 0\,, \\
[e]\, D - [(e + p)\, v_{\mathrm{n}}] = 0\,, \\
[v_{\mathrm{t}}] = 0\,,
\end{gathered} \tag{2}$$

where v_{n} and v_{t} are the velocity components normal and tangential to the detonation front.

Let us introduce additionally the dimensionless coefficients α_k (k = 1, 2) describing the shape of the lateral wall $f(x)$ chosen from some class. For the class of straight lines, the parameter α_k has only one value of α: $f(x) = R[1 + \tan(\alpha)x]$, and for the class of parabolas it has two values: $\alpha_1 = a$, $\alpha_2 = b$: $f(x) = R[1 + x(a + bx)]$.

Let us introduce the dimensionless variables as follows:

$$\begin{gathered}
x \to \frac{x}{R}\,,\quad r \to \frac{r}{R}\,,\quad p \to \frac{p}{p_0}\,,\quad \rho \to \frac{\rho}{\rho_0}\,,\quad u \to \frac{u}{\sqrt{p_0/\rho_0}}\,, \\
L \to l = \frac{L}{R}\,,\quad Q \to q = \frac{Q}{\gamma p_0/\rho_0}\,,\quad t \to \frac{t}{t_0}\,,\quad t_0 = R\sqrt{\frac{\rho_0}{p_0}}\,.
\end{gathered} \tag{3}$$

Below we give the results obtained at γ = 1.4 and q = 10.5.

The flow behind the detonation wave has high values of the thermodynamic parameters, and due to the pressure difference inside and outside the channel a force acting in the axial direction — a thrust — arises. It is described by the function

$$I(t, \alpha_k) = -\int_{\mathrm{w}} (p_{\mathrm{w}} - 1)\, r n_x\, ds = 2\pi\left(\int_0^1 [p(t, 0, r) - 1]\, r dr + \int_0^1 [p(t, x, f(x)) - 1]\, f(x) f'(x)\, dx\right), \tag{4}$$

where p_{w} is the pressure on the channel walls. Here integration of the excess pressure along the wall has been performed. The first term between brackets is the integral over the surface of the closed end of the channel, and the second term is the integral over the lateral surface.

We investigate the values of the mean momentum [44]

$$I_S(l, \alpha_k) = \int_0^{T(l,\alpha_k)} I(t, \alpha_k)\, dt / T(l, \alpha_k) \tag{5}$$

and of the mean specific momentum

$$J(l, \alpha_k) = \int_0^{T(l,\alpha_k)} I(t, \alpha_k)\, dt / V(l, \alpha_k)\,. \tag{6}$$

Here $T(l, \alpha_k)$ is the instant of time at which the detonation wave reaches the edge of the wall with abscissa l and $V(l, \alpha_k)$ is the volume occupied by detonation products at this instant of time.

Self-similar flow. The simplest shape of the wall is a conical surface described by a straight line on the plane x–r: $f(x) = 1 + \tan(\alpha)x$. If the length of the cone generatrix is much larger than the end radius ($L >> R$ or $l >> 1$), then, ignoring the influence on the flow of the closed end of the channel, we can model it by the known self-similar flow behind the spherical detonation wave [45] propagating with a constant velocity D_J in the Chapman–Jouguet regime, and the gas-dynamic parameters of the flow behind the wave depend on the self-simulated variable $\lambda = \sqrt{x^2 + r^2}/(D_J t)$.

In this case, according to (4)–(6), we have

$$I_S(\alpha) = c_1 V^{2/3} \frac{\sin^2 \alpha}{(1 - \cos\alpha)^{2/3}}, \quad J(\alpha) = c_2 \cos^2 \frac{\alpha}{2}.$$

Here c_1 and c_2 are dimensionless constants containing the integrals of the self-simulated pressure distribution along the wall and depending on q.

The mean momentum at a fixed mass of the burnt mixture reaches its maximum at a half-opening angle $\alpha_0 = 60^o$, and the mean specific momentum reaches its maximum in the limit at $\alpha \to 0$. Note that a decrease in the opening of the cone at a fixed mass of the burnt mixture and unlimited sizes leads to an increase in the length of its generatrix, i.e., this limit is very exotic.

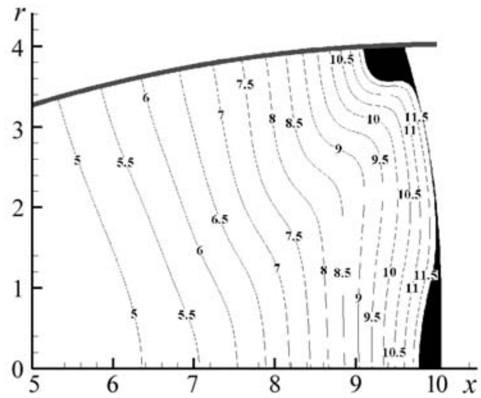
It should be noted that in the case of detonation waves propagating in wedges with half-opening angles α, the value of the mean momentum is proportional to $\sin\alpha/\sqrt{\alpha}$ and reaches its maximum at $\alpha_0 \approx 66.8^o$ (the solution of the equation $\tan\alpha = 2\alpha$). The value of the mean specific momentum is proportional to $\sin\alpha/\alpha$ and reaches its maximum on the self-similar solution at an infinite length corresponding to $\alpha = 0$. Thus, the results obtained for the cone and the wedge agree qualitatively with each other.

The determination of the maximum values of the considered integral characteristics at a curvilinear generatrix of the lateral wall or at comparable sizes of L and R reduces to the solution of the non-self-similar problem. Its thorough investigation is only possible by numerical methods.

Non-self-similar flow. In the present work, to investigate the non-self-similar flow, we use the Godunov finite-difference method with a movable mesh in a domain bounded by fixed solid surfaces and a detonation wave connected to the computational mesh. In the computational procedure, to realize the boundary condition on an explicitly resolved detonation wave, we used the solution of the self-similar discontinuity breakup problem in the detonating gas, and to obtain the real shape of the detonation wave front, we use a special procedure by which the distribution of boundary points connected with the wave is realized; this procedure is based on the investigation of the local behavior of the solution near the detonation wave.

The initial distribution of parameters is given in a narrow zone of width $\delta << R$ near the open end as a solution of the known one-dimensional, self-similar problem on the propagation of an infinitely thin plane detonation wave from the wall. The mesh is expanded according to the position and form of the detonation wave. It is assumed that the detonation wave can be overcompressed and the regime of its propagation on a given segment of the broken line modeling the wave front is determined at each time step in the process of calculation. To this end, the self-similar problem on the discontinuity breakup at the boundary between the detonation products and the combustible mixture is solved. The initial parameters in this problem are the current parameters behind the detonation wave and the parameters before its front. From the solution we determine the detonation-wave velocity and the gas-dynamic parameters behind the front needed to calculate the mass, momentum, and energy fluxes.

The investigation of the behavior of the solution near the wall points to the following features. If for some reason or other the angle between the rear surface of the detonation wave and the wall becomes less than 90^o, i.e., the wave propagates from the wall, then near the wall restructuring of the flow occurs, as a result of which this angle becomes a right angle. At an angle greater than 90^o the intersection point of the detonation wave front and the wall has a velocity higher than the detonation wave velocity. In this case, the detonation wave can become locally overcompressed. As a result, the tendency toward "normalization" of the angle between the detonation wave and the wall is realized. The processes described are illustrated by the results of calculations given below.



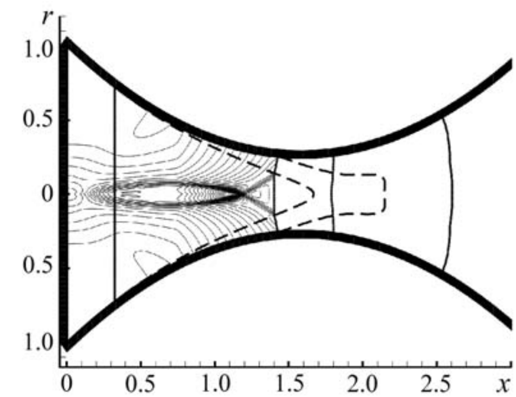


Fig. 7. Contour curves of the shock pressure.

Fig. 8. Shock-wave pattern of the evolution of the process of detonation-wave propagation.

Results of two-dimensional calculations. It should be noted that the computational procedure is effective in the case of walls of various classes. For example, for a divergent nozzle with a parabolic form of the wall generatrix $f(x) = 1 + x(0.6 - 0.3x)$, the calculations have revealed the following features of the flow.

In the process of detonation-wave propagation, the angle between its front and the wall remains greater than 90°, since the inclination of the wall decreases steadily. In the region near the intersection of the wall and the detonation wave, large density and pressure values are attained but the regime of detonation propagation remains Chapman–Jouguet. Near the x axis, a plane section of the wave front is observed. Its presence in the case of the Chapman–Jouguet regime can be justified, having constructed by the Huygens principle a curve of the detonation-wave form at a given instant of time t. This curve is equidistant from the channel end for distance $D_J t$ and consists of two circular arcs and a plane section coinciding in size with the end. The above details of the flow are illustrated by Fig. 7, showing the isobars at some instant of time. The dark region near the detonation wave front shows the behavior of the pressure in the vicinity of the front. The curve bounding this region on the left corresponds to the pressure equal to 11.7. Directly on the detonation wave front it is equal to 14. The isobars in this region start on the solid wall, approach the detonation front, and then depart from it in the vicinity of the symmetry axis. Such behavior of the isobars is due to the fact that behind the convex Chapman–Jouguet detonation fronts the pressure gradients are infinite, and behind the plane fronts they are finite.

As one would expect, the regime of overcompressed detonation is realized in convergent nozzles. The Chapman–Jouguet detonation can go thereby to the overcompressed regime and then go back to the Chapman–Jouguet regime. The angle between the detonation wave front and the wall behind the "neck" is less than 90°, since the inclination of the wall at the point of its intersection with the detonation front increases steadily as the latter propagates.

In the present work, convergent nozzles were used to test the numerical scheme. The results obtained prove the applicability and reliability of the developed and realized numerical procedure. As the calculations have shown, convergence of the channel can lead to the transition of the wave to the regime of overcompressed detonation. However, such nozzles cannot be used in detonation engines by virtue of the presence of a negative thrust at the first stage of detonation propagation because of the nozzle contraction. What is more, because of the channel contraction a reflected shock wave is formed, which is shown in Fig. 8. Here the fine lines show isobars crowding together in the flow behind the detonation wave because of the presence of an internal transverse shock wave which at the place of intersection with the portion of the overcompressed wave and the Jouguet wave forms a triple point. The left dotted line shows the trajectory of the triple point, and the right one — the trajectory of the point of contact of the overcompressed detonation wave and the Chapman–Jouguet wave. The four solid lines depict the detonation wave front at instants of times corresponding (from left to right) to: the initiation of overcompressed detonation near the lateral wall, the initiation of a Chapman–Jouguet wave near the wall, the intermediate stage of transition of the overcompressed wave to a Chapman–Jouguet wave due to the expansion of the latter near the wall, and the formed Chapman–Jouguet wave front. Note that in the narrowest place a configuration of the detonation wave front is observed at which this

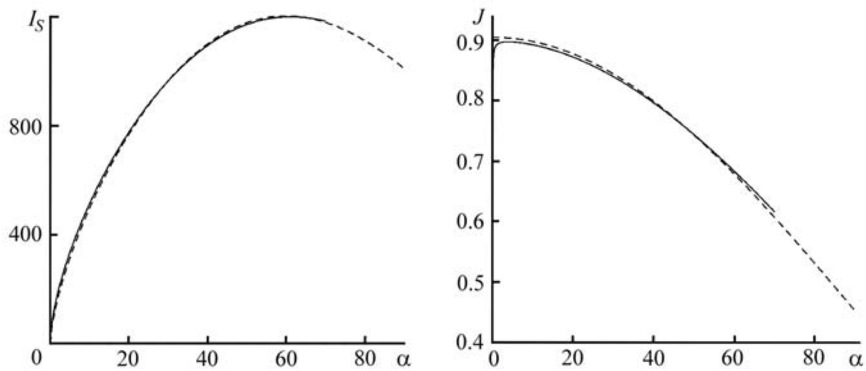


Fig. 9. Mean and mean specific momenta calculated in the two-dimensional approximation versus α.

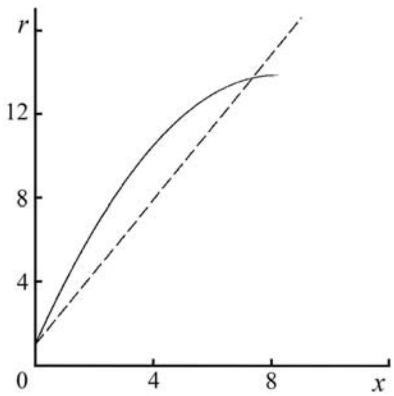


Fig. 10. Optimal parabolic shape (solid line) in comparison with the optimal conical shape (dashed line).

front is formed by the Jouguet ring zone, the intermediate ring of the overcompressed detonation, and the internal disc of the Chapman–Jouguet wave.

Optimization in two-dimensional calculations. In each calculation with a preset shape of the lateral wall, the instantaneous and integral thrust characteristics were determined. The shape of the lateral wall is varied with the aim of independent maximization of the mean momentum I_S and the mean specific momentum J.

The conclusions drawn for the cone in the one-dimensional approximation are confirmed by two-dimensional calculations. For a fixed volume $V = 5000$ and conical walls $f(x) = 1 + \tan(\alpha)x$, the maximum of I_S is attained at $\alpha = 61^{\circ}$, and the maximum of J is attained at $\alpha = 3^{\circ}$. Figure 9 presents the dependences of the mean momentum and the mean specific momentum on α calculated by the parameters of the two-dimensional calculation and by the self-similar solution. The dependences practically coincide. The slight difference is due to the presence in the two-dimensional calculation of the channel end.

Two-dimensional calculations at a fixed volume $V = 5000$ show that the conical wall at $\alpha = 60^{\circ}$ does not correspond to the maximum of the mean momentum, i.e., it is not optimal. In the class of parabolic shapes $f(x) = 1 + x(b + ax)$ containing all cones at $a = 0$ and various b, the maximum of the mean momentum is attained in a nozzle of the following form:

$$f(x) = 1 + x\,(3.15 - 0.193x)\,.$$

Figure 10 shows the plot of this function, as well as, for comparison, the plot of the function $1 + \tan(\pi/3)x$ at which in the class of conical surfaces the maximum value of the mean momentum is attained. The increase in the mean momentum compared to the cone is 8% and is due to the above-mentioned increase in the pressure near the wall because of the decrease in its inclination.

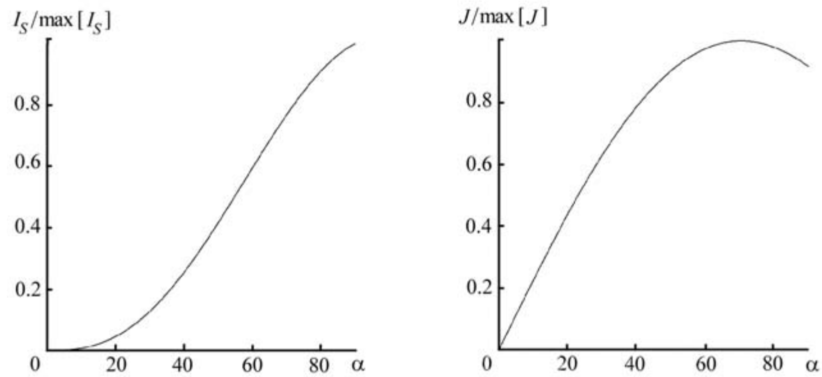


Fig. 11. Mean and mean specific momenta vs. α for the case of instantaneous explosion near the channel end.

The investigations conducted for nonparabolic shapes shows that the increase in the mean momentum is no more than 2%.

An increase in the mean specific momentum in the case of conical surfaces is attained due to an increase in the generatrix length. For instance, at a fixed volume $V = 5000$ the maximum is attained at a half-opening angle $\alpha = 3^{\circ}$ and a length $l = 100$ exceeding the end radius by two orders of magnitude. In this connection, it is very problematic to use such a nozzle in real engines. Therefore, the mean specific momentum was maximized for monotonically increasing parabolic functions $f(x)$ and a fixed length of the channel $l = 5$. The optimal shape of the lateral wall in this case is described by the function

$$f(x) = 1 + x\,(0.0 + 0.061x)\,.$$

Propagation of the blast shock wave in the channel. The integral thrust characteristics were also maximized for the case of instantaneous explosion in an inert medium near the channel end. The explosion is modeled by a homogeneous instantaneous increase in the internal energy of the gas in a narrow zone of width $\delta << R$ near the closed end of the channel, as a result of which the energy in this zone increases by E_0 and a blast shock wave propagates along the channel. The flow is investigated up to the moment the wave leaves the channel. Dedimensionalization is carried out analogously to the case of detonation. The dimensionless blast energy is defined as $e_0 = E_0/(p_0R^3)$. For a conical lateral wall with a large length of the generatrix, $l >> 1$, and at a fairly high energy $e_0 >> V/(\gamma - 1)$, where V is the volume of the region behind the shock wave, at the instant the wave leaves the channel, the flow behind the shock wave can be described with the use of the known self-similar solution of the problem on the strong spherical point explosion. The gas-dynamic parameters of such flow depend on one self-simulated variable:

$$\lambda = \frac{\sqrt{x^2 + r^2}}{\left(\dfrac{E_0}{\rho_0}\right)^{1/5} t^{2/5}}\,.$$

Integration of the pressure along the wall and with respect to time gives values of the mean momentum and the mean specific momentum as a function of the half-opening angle of the cone (see Fig. 11):

$$I_S(\alpha) = c_1 \frac{e_0}{V^{1/3}} \sin^2\alpha\,(1 - \cos\alpha)^{1/3}\,, \quad J(\alpha) = c_2 \sqrt{\frac{e_0}{V}}\, \frac{\sin^2\alpha}{(1 - \cos\alpha)^{1/2}}\,.$$

Here c_1 and c_2 are dimensionless constants containing integrals of the self-similar pressure distribution along the wall depending on e_0.

The investigation has shown that the mean momentum at a fixed volume of the gas behind the shock wave increases with increasing angle α. The mean specific momentum reaches its maximum at a half-opening

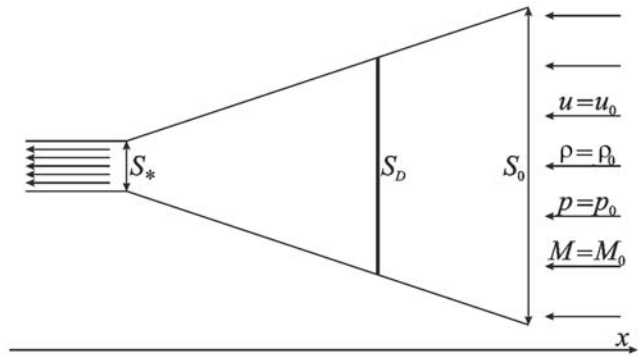


Fig. 12. Scheme illustrating the formulation of the problem.

angle α = arccos (1/3) $\approx 70.5^{\circ}$. Note that in the case of a wedge, the self-similar solution describing the cylindrical point explosion gives the maximum of the mean specific momentum at $\alpha \approx 66.8^{\circ}$.

The conclusions drawn for the cone are also confirmed in two-dimensional calculations. For instance, for a fixed blast energy $e_0 = 100V/(\gamma - 1)$ and a volume $V = 5000$ in a cone with a generatrix $f(x) = 1 + \tan(\alpha)x$, the maximum of J is attained at $\alpha = 71^{\circ}$.

Thus, by means of the self-similar solutions describing the flows behind the detonation and blast waves in a cone and a wedge with different opening angles, the angles corresponding to the maximum of the mean momentum have been found. In the case of detonation for the cone, it equals 60°, and for the wedge — 66.8°. Under propagation of the blast shock wave, the mean specific momentum reaches its maximum at an angle arccos (1/3) for the cone and 66.8° for the wedge. By numerical calculations of two-dimensional flows in axisymmetic channels, the optimal shapes of the lateral wall corresponding to the maximum of the mean momentum and the mean specific momentum have been found.

Stabilization of the Detonation Wave in a Variable-Cross-Section Channel. In this section, the model of an infinitely thin detonation wave is used to analyze the possibility of stabilizing the detonation in a variable-cross-section channel that can be regarded as a combustion chamber. Note that supersonic flows with a shock wave in a variable-cross-section channel were studied in detail in [46–48]. An interesting and practically important question was the question on the possibility of the existence of such a flow at which the detonation wave is immobile with respect to the channel walls. Let us call such a wave the stabilized wave and the determination of the conditions for its realization the stabilization.

Formulation of the problem. The flow with a detonation wave in a supersonic flow of a combustible gaseous mixture in a variable-cross-section channel is considered in the quasi-one-dimensional approximation. The shape of the channel is given by the dependence of the cross-section area on the single, in the problem, Cartesian coordinate $S(x)$, and the gas dynamics equations in the region of continuous change in the parameters have the form

$$\frac{\partial (\rho S)}{\partial t} + \frac{\partial (\rho u S)}{\partial x} = 0 ,$$

$$\frac{\partial (\rho u S)}{\partial t} + \frac{\partial (p + \rho u^2) S}{\partial x} = p \frac{dS}{dx} , \tag{7}$$

$$\frac{\partial \rho (\varepsilon + u^2/2) S}{\partial t} + \frac{\partial [\rho u (\varepsilon + u^2/2) + pu] S}{\partial x} = 0 .$$

Let the x axis be directed counter to the flow velocity, and the function $S(x)$ be nondecreasing (see Fig. 12).

The incident flow parameters are given in some cross section with area S_0 and denoted by 0: pressure — p_0, density — ρ_0, velocity — u_0, Mach number — M_0. In the channel under consideration, the stationary supersonic flow stagnates or maintains its velocity.

To elucidate the question on the existence of a flow with an immobile detonation wave, let us consider the class of stationary flows and make use of the following known relations for quasi-one-dimensional steady flows of the perfect gas following from (7):

$$\mathrm{M} = q^{-1}\left(q\,(\mathrm{M}_0)\,\frac{S_0}{S}\right), \quad q\,(\mathrm{M}) = \frac{\mathrm{M}}{\left(\dfrac{2+(\gamma-1)\,\mathrm{M}^2}{\gamma+1}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}},$$

$$u = u_0\,\frac{\mathrm{M}}{\mathrm{M}_0}\left(\frac{2+(\gamma-1)\,\mathrm{M}_0^2}{2+(\gamma-1)\,\mathrm{M}^2}\right)^{\frac{1}{2}},$$

$$\rho = \rho_0\left(\frac{2+(\gamma-1)\,\mathrm{M}_0^2}{2+(\gamma-1)\,\mathrm{M}^2}\right)^{\frac{1}{\gamma-1}}, \tag{8}$$

$$p = p_0\left(\frac{2+(\gamma-1)\,\mathrm{M}_0^2}{2+(\gamma-1)\,\mathrm{M}^2}\right)^{\frac{\gamma}{\gamma-1}}.$$

On the stationary detonation wave, the mass, momentum, and energy conservation laws of the form

$$\rho_2 u_2 = p_1 u_1\,,$$

$$p_2 + \rho_2 u_2^2 = p_1 + \rho_1 u_1^2\,, \tag{9}$$

$$\frac{u_2^2}{2} + \frac{\gamma p_2}{(\gamma-1)\,\rho_2} + Q = \frac{u_1^2}{2} + \frac{\gamma p_1}{(\gamma-1)\,\rho_1}$$

should be fulfilled. Here Q is the specific heat release, the index 1 corresponds to the region before the wave front, and 2 — behind it.

The detonation wave can be in the cross-section area $S(x)$, where the gas velocity before it determined by (8) as $u_1 = u_1(\mathrm{M}(S))$ is not lower than the velocity of the Chapman–Jouguet wave, i.e., $u_1 \geq D_J$. The equivalent inequality expressed in terms of the known values before the wave is obtained by transforming system (9) to the quadratic equation for ρ_1/ρ_2 for the point of intersection of the Michelson line with a Hugoniot adiabat:

$$(\gamma+1)\left(\frac{\rho_1}{\rho_2}\right)^2 - 2\left(\gamma + \frac{1}{\mathrm{M}_1^2}\right)\frac{\rho_1}{\rho_2} + \frac{2}{\mathrm{M}_1^2} + (\gamma-1)\left(\frac{2Q}{u_1^2} + 1\right) = 0\,. \tag{10}$$

The discriminant of Eq. (10) should be nonnegative:

$$\left(1 - \frac{1}{\mathrm{M}_1^2}\right)^2 - (\gamma^2-1)\,\frac{2Q}{u_1^2} \geq 0\,,$$

i.e., the inequality

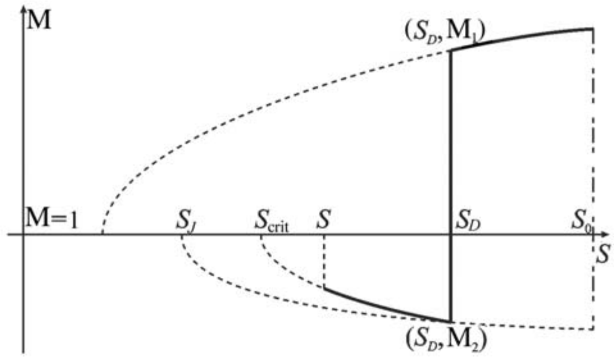


Fig. 13. Diagram of the parameters S, M.

$$\frac{1}{2(\gamma^2-1)}\left(1-\frac{1}{M_1^2}\right)^2 u_1^2 \geq Q \tag{11}$$

should hold.

In the general case, at a strict inequality in [11], the stationary detonation wave is overcompressed, which corresponds to the smaller root of Eq. (10). It should be noted that the flow behind a wave with $M < 1$ will accelerate downstream due to the decrease in the cross-section area, and the Mach number can reach unity.

The Chapman–Jouguet regime is only possible in the case of equality in (11). In this case, the detonation wave propagates over detonation products with the velocity of sound, i.e., directly behind its front there is a sonic line. Behind such a wave the channel cannot converge because of the impossibility of realizing both a subsonic and a hypersonic flow even if the existence of additional shocks inside the flow region is assumed. Thus, stationary flow with an immobile Chapman–Jouguet detonation wave is only possible when the detonation wave is in the minimal cross section S_*.

Analysis of the flow with an overcompressed detonation wave shows that since the flow behind it is subsonic, the Mach number increases as the area of the cross section decreases downstream. As for the Chapman–Jouguet wave, in the case of an overcompressed detonation wave, the flow behind it can become sonic only in the minimal cross section.

For further analysis of various flow patterns, let us introduce into consideration the plane of parameters S, M on which we will mark the points corresponding to all possible coordinates x (Fig. 13).

From conditions (9), the values of the flow parameters behind the wave front are given as follows:

$$M_2 = \sqrt{\frac{1}{\frac{1}{M_1^2 k(M_1)} + \gamma\left(\frac{1}{k(M_1)} - 1\right)}},$$

$$a_2 = a_1\sqrt{k(M_1)\,(1-\gamma M_1^2\,(k(M_1)-1))},$$

$$u_2 = u_1 k(M_1), \quad \rho_2 = \frac{\rho_1}{k(M_1)}, \quad p_2 = p_1\,(1-\gamma M_1^2\,(k(M_1)-1)), \tag{12}$$

where

$$k(M_1) = \frac{\gamma + \frac{1}{M_1^2} - \sqrt{\left(1-\frac{1}{M_1^2}\right)^2 - (\gamma^2-1)\frac{2Q}{u_1^2}}}{\gamma+1} \leq 1.$$

Relations (12) make it possible, for a given position of the detonation wave, to represent the values of the parameters u, ρ, p, and M as functions of S up to the minimal cross section of the area S_{crit} determined from the condition $\mathrm{M} = 1$.

To the flow regions before the detonation wave positioned in a cross section with the coordinate x_D and area S_D will correspond a section of the curve $S > S_D$, $\mathrm{M} > 1$ described by the expression for M in (8). At $S = S_D$ the flow becomes subsonic (or sonic at $S_D = S_J$), and the Mach number changes stepwise from $\mathrm{M}_1 > 1$ to $\mathrm{M}_2 \le 1$ according to the expression for M in [11]. Varying the value of S_D, we can obtain various points (S_D, M_2) under the $\mathrm{M} = 1$ axis lying on the curve shown in Fig. 13 by a dotted line and ending by the point ($S = S_J$, $\mathrm{M} = 1$). As the channel converges downstream, the point (S, M) will move along the section of the curve $S < S_D$ up to the point $S = S_{\mathrm{crit}}$, where $\mathrm{M} = 1$. If the minimal outlet cross section $S_* > S_{\mathrm{crit}}$, then the section of the curve in the plane (S, M) corresponding to the subsonic flow behind the detonation wave will end before the point ($S = S_{\mathrm{crit}}$, $\mathrm{M} = 1$) and the subsonic flow parameters will remain constant along the whole of the section with $S = S_*$. From this it follows that the stationary detonation wave can be positioned only in cross-sections with an area S_D larger than some critical S_J, and the minimum value of the cross section area S_* should be larger than the other critical value of S_{crit}:

$$S_J \le S_{\mathrm{crit}} \le S_* \le S_D\,. \tag{13}$$

Note that the critical value of S_{crit} depends on both the value of Q and the value of S_D, and, therefore, also on the position of the detonation wave x_D. The value of S_J is only determined by the specific heat release Q, and at $Q = 0$ the end points of the dotted lines in Fig. 13 coincide. If the shape of the channel $S(x)$ and the Q value are fixed, then the values of S_J, S_* will be strictly definite. To various values of the parameters S_D there correspond therewith various values of S_{crit}. The requirement for the inequality $S_{\mathrm{crit}} \le S_*$ to hold leads to the inequality $S_J \le S_* \le S_D \le S_{D_{\mathrm{crit}}}$ equivalent to (13) and imposing restrictions on the position of the stationary detonation wave.

We emphasize that analysis of the curves on the plane (S, M) permits determining the range of possible positions of the stationary detonation wave in a channel with a given law of change in the cross-section area. In particular, at a rather small minimal area of the cross section S_* in the channel a stationary flow with a detonation wave is impossible. To illustrate the possibilities of the described approach in analyzing the existence of a stationary flow, consider the channel represented in Fig. 12. Let a certain value of Q be given. Then we can state at once that there will be no standard flow if the minimal cross-section area is less than the value of S_J determined by formulas (12) at $\mathrm{M}_2 = 1$. Otherwise a stationary flow is possible but only when the detonation wave is on the left of the critical cross section of area $S_{D_{\mathrm{crit}}}$.

Thus, it has been shown that when inequality (13) holds, a stationary flow with a detonation wave immobile with respect to the channel walls is possible; however, the question on the stability of the solution remains to be elucidated.

Numerical stability check. The flow stability was investigated numerically by S. K. Godunov finite-difference method [43] on a movable computational mesh. In so doing, the detonation wave was resolved as the right boundary of the computational mesh and its parameters were determined from the solution of the self-similar problem on the breakup of discontinuity separating the detonation products and the combustible mixture.

It should be noted that in the above-considered problem the flow in the outlet cross section is subsonic. However, in the case where at the outlet from the channel the gas velocity reaches the velocity of sound, the flow behind a cross section of area S_* can be accelerated to the supersonic velocity in a Laval nozzle. In so doing, the diverging part of the nozzle creates a thrust, which permits considering the whole of the structure as a power unit. Moreover, the presence at the channel outlet of an area with a supersonic flow eliminates the necessity of using in the numerical investigation special nonreflecting boundary conditions. In the calculations made, the left boundary of the computational domain was a fixed outlet cross section of the Laval nozzle, in which the gas-dynamic parameters were taken to be equal to the known instantaneous values of the parameters of this supersonic flow. As the investigation has shown, to obtain complete information on the stability, it is necessary to complicate the shape of the channel. To this end, we added to the cross-section of the channel of area S_0 a section with a monotonically downstream increasing area into which a homogeneous supersonic flow with parameters p_0', ρ_0', u_0', and M_0' flowed through the inlet cross section S_0. These parameters were given so that at $S = S_0$ the gas had the same values of the parameters as at the

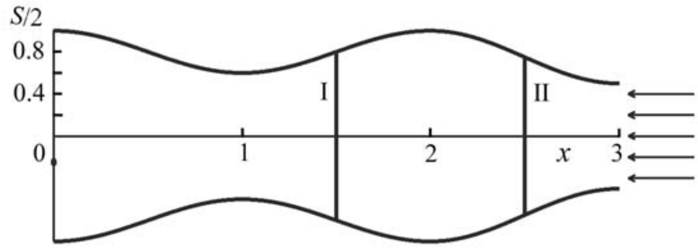


Fig. 14. Scheme of the channel of complex shape: I, unstable position of the detonation wave; II, stable position.

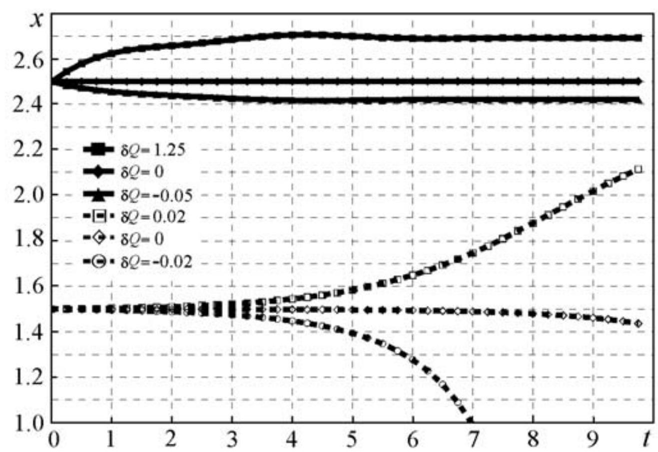


Fig. 15. Diagrams of the dependence of the wave coordinate x on the time t.

channel inlet in the above analysis of the existence of a stationary flow: p_0, ρ_0, u_0, and M_0. The area of the channel of complex shape was given in the calculations as (see Fig. 14)

$$S(x) = \begin{cases} 1.2 + 0.8\cos^2(\pi x/2)\,, & 0 < x < 2\,; \\ 1 + \cos^2(\pi x/2)\,, & 2 < x < 3\,, \end{cases}$$

where the constants were obtained on the condition that all sizes refer to a third of the channel length.

Below we present the results of investigation of the stability for two characteristic cases where the wave was located at points $x = 1.5$ and 2.5 in the sections converging and diverging in the direction of the flow, respectively.

Calculations were performed at a value of the adiabatic index $\gamma = 1.3$. To obtain the initial distribution of the parameters necessary in the numerical investigation, we used the analytical solution defined by relations (8), (12) at a given constant value of the specific heat release $Q = Q_0$. Disturbance of the flow was realized by a small change in the value of the specific heat release on the detonation wave $Q = Q_0 + \delta Q$. It should be noted that all results do not depend qualitatively, and with an accuracy to the scales also quantitatively, on the actual value of Q_0, since the entire analysis can be carried out at any fixed value of the heat release on the detonation wave. Because of this, the entire numerical investigation was conducted in dimensionless variables in which $Q_0 = 1$.

Figure 15 shows in dimensionless variables the plots of the detonation-wave coordinate x as a function of the time t for the two above-mentioned initial positions of the wave and various values of δQ. According to the calculations, in the converging section the wave is unstable. Even at $\delta Q = 0$ it gradually, without stopping, departs from its initial position because of the disturbances due to the computational errors at the level of computer-based accuracy. It turned out that the sign of δQ influences the direction of propagation of the detonation wave, which is well illustrated by Fig. 15, where the dotted lines correspond to the unstable case for two small values of δQ of different signs and the value of $\delta Q = 0$. The results obtained suggest that stabilization of the detonation wave in the combustion chamber is possible due to the variable heat release that can be realized by varying the composition of the combustible mixture.

Evidently, practical implementation of the given idea is connected with monitoring the behavior of the detonation wave and the availability of positive feedback.

The calculations for the point $x = 2.5$ show that in the downstream diverging section of the channel the detonation wave appears to be stable. At $\delta Q = 0$ the wave does not change its position relative to the channel because of the disturbances connected with round-off errors, and at $\delta Q \neq 0$ it shifts from its initial position, with the shift direction depending on the sign of δQ. At $\delta Q > 0$ the wave shifts upstream, and at $\delta Q < 0$ — downstream. Interestingly, the detonation wave, striving to occupy a certain position relative to the channel, may first jump over it.

Thus, we have developed an analytic method by which in special coordinates the conditions for the existence of a stationary flow behind the detonation wave have been investigated. The possibility of detonation-wave stabilization has been shown analytically and by numerical experiments. Analysis of the stationary-state resistance towards small perturbations of the heat release permitting selection of the realized flow structure has been carried out.

Initiation of Detonation by a Ledge in a Supersonic Flow. Attempts to use detonation in engines and other power plants are faced with a number of problems. The chief problems are the excitation of detonation and its stabilization within the limits of the combustion chamber. In this connection, we consider a problem on the initiation of detonation in a supersonic flow of a stoichiometric mixture filling, partly or completely, a plane channel in the transverse direction. Initiation of the flow occurs due to a ledge or a wall blocking the channel. The investigation is conducted within the framework of single-stage combustion kinetics. The initiation of detonation in a layer under the conditions of an unbounded space and a stagnant medium was studied experimentally in [49].

Formulation of the problem. Consider a plane channel of finite length consisting of two sections of constant different widths and a section connecting them whose lower boundary is a quarter of the circle. The right and the left ends of the channel are open. It is assumed that from the right a supersonic flow will enter the channel and through the left end outflow into a vessel of large volume whose walls are impenetrable occurs. Let the channel and the vessel be filled with still air with pressure p_0, density ρ_0, and temperature T_0, and at $t = 0$ through the right end of the channel a supersonic flow of air or a combustible mixture with the same parameters p_0 and T_0 with constant velocity begins to enter the channel. As a result of this, a nonstationary flow is initiated in the channel. Let us introduce a Cartesian coordinate system whose origin is on the left end of the channel, and the lower wall of the right section corresponds to $y = 0$. We shall determine the geometric parameters in terms of the sizes of three sections in the direction of the x axis — l_1, r, and l_2. In so doing, r is the radius of the circle a quarter of which is the lower curvilinear wall of the second section. Denoting the width of the third section by h, we obtain that the width of the second section is equal to $h - r$. Thus, the considered geometry of the channel is defined by four independent dimensional quantities — l_1, r, l_2, and h. By virtue of the foregoing, in the accepted coordinate system, to the left end of the channel there corresponds $x = 0$, to the right end $x = l_1 + r + l_2$, to the center of the circle $x = l_1$ and $y = 0$, and to the upper wall $y = h$. Then the lower wall can be described by the following relation:

$$y(x) = \begin{cases} r, & 0 < x < l_1; \\ \sqrt{r^2 - (x - l_1)^2}, & l_1 < x < l_1 + r; \\ 0, & l_1 + r < x < l_1 + r + l_2. \end{cases}$$

The vessel into which the gas from the channel flows has the shape of a triangle with sizes along the x axis $150h$ and along the y axis $100h$. The hole in the vessel is in the middle of the side of the triangle corresponding to $x = 0$.

It should be noted that the presence of a vessel with the above parameters does not influence the flow in the channel in the considered time, including the process of steadying of the air flow in the channel and the subsequent development of detonation in the combustible mixture.

Let us introduce dimensionless diagnostic variables and functions, taking for the basic dimensional quantities the width of the third section of the channel h, the pressure p_0, and the velocity of sound a_0 in the incident flow. Then $t_0 = h/a_0$ will be used as a time unit, and the quantity $\rho_0 = p_0/(a_0^2)$ will be used as a unit of density.

The character of the flow and its dimensionless parameters as functions of the dimensionless coordinates and time will depend on the Mach number of the incident flow $M = U/a_0$. Subsequently, the Mach number is, as a rule, the only variable parameter in the problem, and the other dimensionless parameters (e.g., those determining the geome-

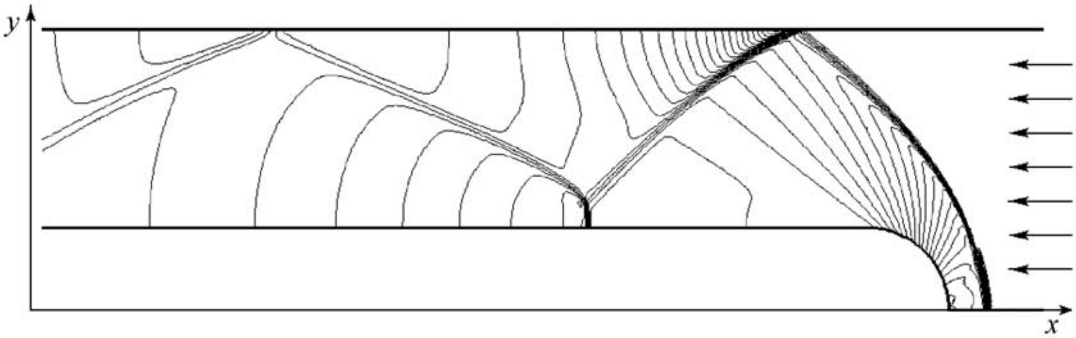


Fig. 16. Isobars in the stationary supersonic air flow for the Mach number M = 4.2.

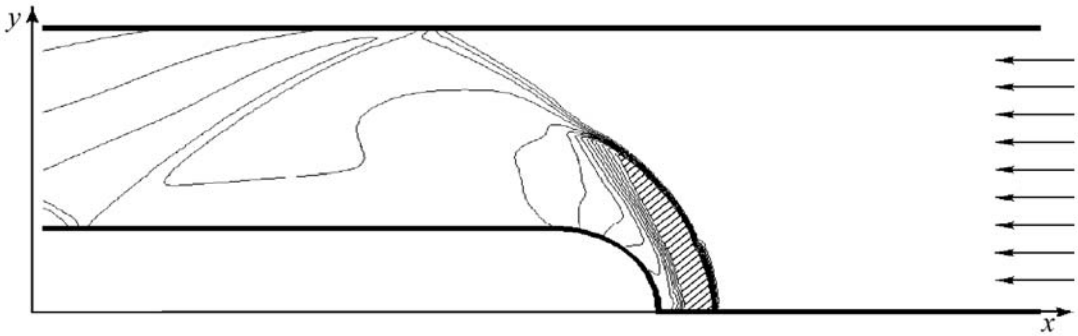


Fig. 17. Isotherms by ignition of the combustible mixture in the channel with a ledge for M = 6.

try of the flow region) are fixed. In those cases where the influence of the values of the other dimensionless parameters on the character of the flow are investigated, we give only the qualitative dependence on them.

The calculation is carried out in a finite domain bounded by the solid walls of the channel and of the rectangular vessel, as well as by the cross section in which a supersonic flow is given. The latter condition guarantees the correctness of the problem formulation.

Below we present the results obtained at p_0 = 1 atm and T_0 = 20°C, l_1 = 0.5 m, r = 0.05 m, and l_2 = 1.45 m.

Steadying of the air flow and combustion mixture inflow throughout the cross section. The calculations of the inert-medium flow at various values of the Mach number of the incident supersonic flow M have shown the following. For a rather small M, stationary flow is impossible because of the channel "choking" effect. In the channel, in front of a ledge, a shock wave propagating upstream to the inlet cross section is thereby formed. The critical Mach number M_0, beginning with which a stationary flow is formed, according to the calculations, is M_0 = 2.8. At $M > M_0$ in the channel a stationary flow with a departed shock wave situated the closer to the ledge the higher the flow velocity is initiated. As an example, Figure 16 pertaining to the case of a stationary air flow, clearly shows a complex wave pattern of the flow. The departed shock wave is regularly reflected from the upper wall, whereas the interaction of the reflected wave with the lower wall leads to a Mach configuration.

Let the condition $M > M_0$ be fulfilled and beginning with some instant of time t_* the flow in the channel be practically stationary. Let, at this instant of time, a homogeneous stoichiometric propane–air mixture with parameters U, p_0, ρ_0, and T_0 begin to flow into the channel through the entire cross section.

Calculations show that when a combustible mixture flows into the channel through the entire cross section, its parameters upon interacting with the departed shock wave increase to such values that the mixture can ignite and detonate. If the Mach number of the incident flow is smaller than the critical number M_* = 3.7, then no detonation occurs but normal combustion is possible [50, 51]. In any case, because of the density difference between the air and the combustible mixture, the flow transforms and the departed shock wave changes its position. At $M > M_*$ the shock wave transforms to a detonation wave. Figure 17 shows the isotherms for M = 6 when the mixture ignites near the lower wall. The shaded portion shows the high-temperature ignition region. At a Mach number of the flow somewhat

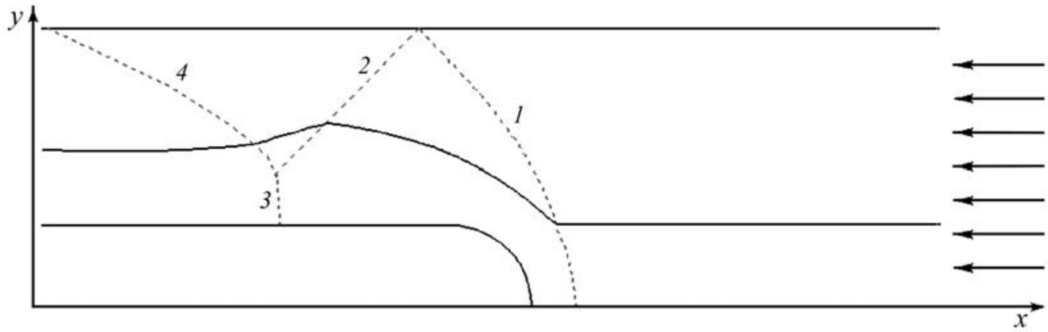


Fig. 18. Combustible mixture jet without ignition: 1) departed wave; 2) shock wave reflected from the upper wall; 3) Mach leg; 4) shock wave reflected from the lower wall.

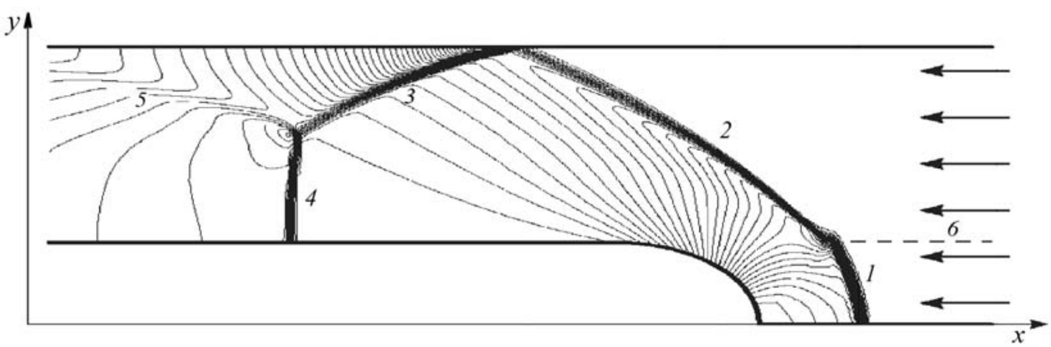


Fig. 19. Isobars at stationary detonation for M = 7.6: 1) detonation wave; 2) air shock wave adjoining the detonation wave; 3) shock wave reflected from the upper wall; 4) Mach leg; 5) shock wave reflected from the lower wall; 6) jet boundary.

higher than the critical value of M_*, the ignition conditions for the combustible mixture can arise not near the critical point on the ledge but behind the shock wave near the upper wall of the channel. Whatever the place of ignition, an identical flow pattern with a detonation wave is formed.

If $M_* < M < M_{**} = 9$, then a detonation wave propagating upstream to the inlet cross section of the channel is initiated, and the flow is nonstationary therewith because of the "choking" caused by the heat input. At $M > M_{**}$ an overcompressed detonation wave is formed and stabilizes near the ledge in the stationary flow similarly to the departed shock wave. The value of the Mach number M_* is determined by the ledge sizes, and the M_{**} value depends on the area ratio between the inlet and outlet cross sections. According to the calculations, as the ratio of the outlet cross-section area to the inlet one decreases to a certain critical value, none of the above flow regimes disappears, but the values of M_* and M_{**} change. If the ratio is smaller than the critical one, then $M_* = M_{**}$, which means the absence of stationary detonation at any flow velocity.

Inflow of the combustible mixture jet. A more complex flow pattern is observed when the combustible mixture flows in not through the entire cross section but only through its part adjoining the lower wall of the channel, i.e., in the form of a plane jet. Below we present the results pertaining to the case where the jet thickness is equal to the thickness of the ledge. Here, there also exists a minimum value of the Mach number at which detonation is initiated, and it coincides with M_* found earlier. From this it follows that this Mach number is determined by the ledge height. At a Mach number smaller than the critical one, the combustible mixture jet passes over the ledge without igniting. This is well seen in Fig. 18, where the solid line shows the boundary of the propane–air jet, and the dotted lines show the departed shock wave 1, the shock wave reflected from the upper wall 2, the Mach leg 3, and the shock wave reflected from the lower wall 4. Waves 2, 3, and 4 form a classical Mach configuration. One's attention is caught by the nonmonotonic character of the change in the jet boundary ordinate. It reaches the maximum value, and the bound-

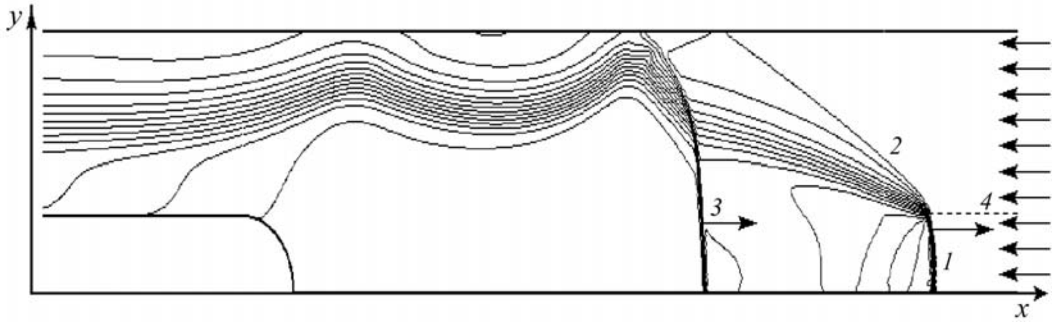


Fig. 20. Isotherms in the propagation of the stationary wave complex at M = 5: 1) detonation wave; 2) air shock wave adjoining the detonation wave; 3) shock wave propagating behind the bow front; 4) jet boundary.

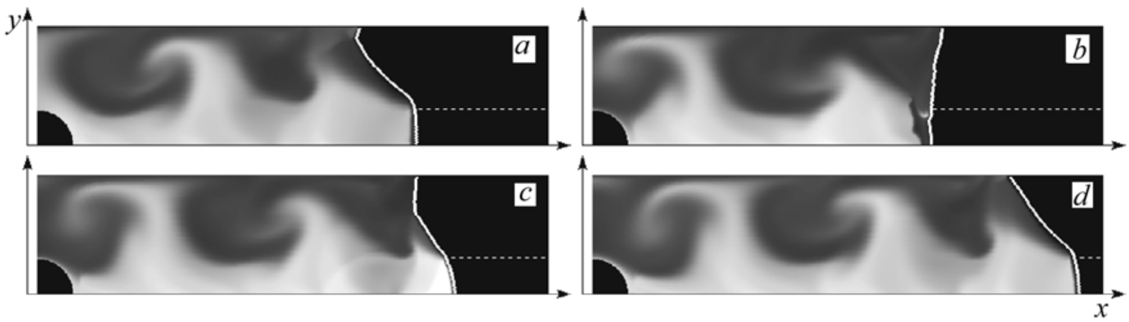


Fig. 21. Wave fronts and temperature fields at various instants of time illustrating the phases of galloping detonation in the ledge channel.

ary experiences a bend at the point of its intersection with the shock wave 2. Downstream the boundary ordinate decreases monotonically and there is a second bend of the jet boundary at the point of its intersection with the wave 4.

In the case under consideration, as for the homogeneous mixture, there exists a critical value of the Mach number $M_{***} = 5.3$, beginning with which the detonation wave stabilizes and the flow throughout the channel becomes stationary. In so doing, the shock wave in the air abuts on the detonation wave in the combustible layer (Fig. 19). The isobars presented it make it possible to see the complex wave pattern of the stationary flow formed.

Depending on the M value, the reflection of the wave 2 from the upper wall can be regular (Fig. 19) or Mach. Mach reflection in the stationary flow is realized in a narrow range of Mach numbers beginning from M_{***}. Of particular interest is the case where $M_* < M < M_{***}$, where there is no stationary flow. Calculations show that at $M_{**} < M < M_{***}$ ($M_{**} = 4.25$) the detonation wave propagates towards the inlet cross section and the complex consisting of a shock wave in the air and a detonation wave retains its form in the process of propagation. In so doing, the wave pattern of the flow is similar to the stationary case at $M > M_{***}$ (Fig. 20). Behind the bow-wave complex consisting of a shock wave and a detonation wave abutting on each other, a combination of shock waves in the detonation products and in the air, whose presence us due to the "choking" of the flow, is clearly seen. It is essential that this configuration moves towards the inlet cross section of the channel, as does the bow shock, but with a lower velocity.

At $M_* < M < M_{**}$, a new, previously unknown regime of detonation in a supersonic flow, namely, galloping detonation in the layer, is realized. In this case, the detonation wave tends to be drifted by the flow or stabilize in a certain position. However, the shock wave formed due to the flow "choking" effect leads the detonation wave and penetrates into the fresh combustible mixture. It heats the portion of the layer before the detonation wave, which after some time bursts into flame and the detonation moves forward in jumps leading the shock front that promoted its jump-like advance. In the absence of a shock leader the detonation wave is drifted by the flow or stops, and the above-mentioned shock wave leads it again and the process is periodically repeated.

Figure 21 shows the pattern of the wave fronts against the background of the temperature field at several instants of time illustrating the main phases of galloping detonation. The white solid line shows the compression shock that separates the disturbed flow region from the incident supersonic flow. The dotted line shows the jet boundary.

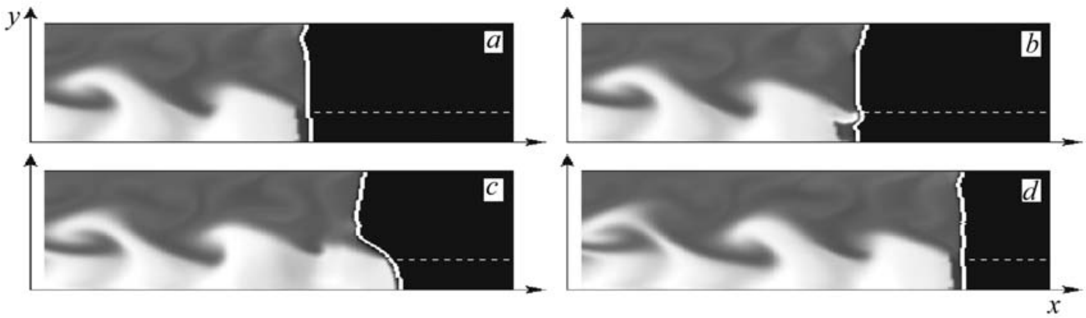


Fig. 22. Temperature fields near the galloping detonation wave.

The lightest region represents high-temperature detonation products. The lesser light region behind the compression shock contains shock-heated air. They are separated by the contact surface. Figure 21a corresponds to the instant of time at which the air shock wave lagging behind the detonation wave begins to catch up with it. In Fig. 21b, it outstrips the detonation wave due to the fact that the latter is drifted by the flow more intensively. In so doing, at the air–combustible mixture interface behind the bow shock front a small ignition zone (the dark region between the white line and the light zone) is seen. Unlike the known "explosion in explosion" phenomenon observed in a stagnant homogeneous combustible mixture at which a new detonation front is formed in the flow behind the main front, in the given case, because of the fuel inflow not through the entire cross section of the channel but only near the lower wall, "leading explosion" takes place. Figure 21c demonstrates the wave pattern of the flow when the mixture heated by the shock wave has completely detonated and the detonation front has outstripped the shock wave in the air. Figure 21d shows the temperature field at the end of the galloping period. The wave front and the temperature field fully correspond to Fig. 21a but the whole of the pattern has moved to the inlet cross section of the channel.

The above-described behavior of the wave is associated with the finite rate of the reactions and is not observed in using the model of an infinitely thin detonation wave. Calculations have shown that with increasing reaction rate (constant A value) the oscillation frequency increases, and the oscillation amplitude of the shock wave with respect to the detonation wave decreases to zero. The key role in the process is played by the time scale of the chemical reaction $\tau \sim 1/A$. Using the parameter τ for dimensionalization in the considered problem, it can easily be obtained that the time and the linear parameters are proportional to τ. Having the data on a particular calculation, we can use them for various values of A by varying proportionally the linear sizes of the channel l and the time according to the equality lA = const.

The critical values of M_*, M_{**}, and M_{***}, as in the case of the combustible mixture inflow along the entire cross section, depend on the sizes of the channel, the ledge, and the jet. The presence of the regime of galloping detonation is closely connected thereby with the boundedness of the channel in the transverse direction. An increase in the channel width at a constant thickness of the jet and at the same size of the ledge leads to a decrease in the value of M_{***} to the value of M_*, as a result of which the regime of galloping detonation disappears.

The term "galloping detonation" appeared earlier in connection with the regime of detonation on the limits of its existence as to the concentrations and pressure that had been observed experimentally in a homogeneous stagnant medium. According to the experiments, with decreasing pressure in an undisturbed mixture the real cellular detonation transforms to spin detonation and then goes to the galloping regime [52].

Analogy with a piston. In the considered problem, the interaction of the supersonic gas flow with the ledge is determining. In the coordinate system associated with the flow, the ledge can be regarded as a piston or a body advancing on the stagnant layer of the combustible mixture. If the ledge blocks the entire channel cross section, then it sets both the combustible and the inert medium in motion. Returning to the previous coordinate system, we obtain a problem on the supersonic flow incident on a fixed wall. According to the calculations, as before in the above-considered problem, there exists a minimum Mach number of the incident flow M_*, beginning from which detonation is formed. At $M > M_*$, depending on the M value, two detonation regimes are observed. If $M < M_*$, then galloping detonation is realized and the frequency of the oscillation process increases with increasing Mach number of the flow until the latter reaches the critical value of M_{**}. At $M > M_{**}$ a wave pattern with a stationary complex similar to that described above is formed. Figure 22 shows the temperature fields for four characteristic instants of time disclosing the

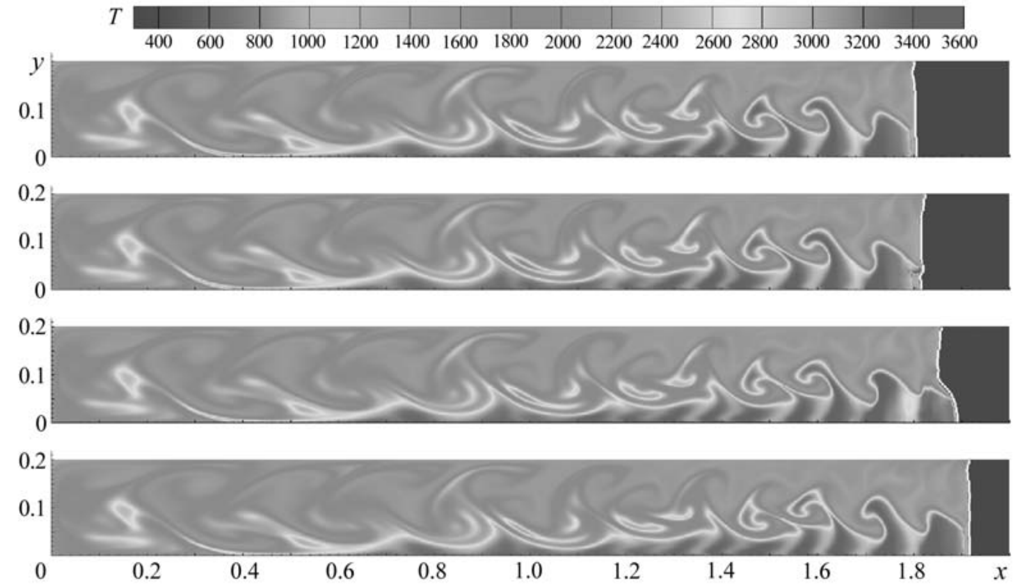


Fig. 23. Temperature fields under galloping detonation in the jet of the combustible mixture incident on the solid wall.

propagation mechanism of galloping layered detonation. As before, the dotted line shows the jet boundary. From Fig. 22a it is seen that the shock wave leads the detonation wave propagating across the layer of the combustible mixture. At the next instant of time (Fig. 22b), the mixture begins to ignite near the upper boundary of the layer. Figure 22c shows the jump-like advance of the detonation wave. The last image replicates the first one and is the start of a new cycle (Fig. 22d).

Figure 23 presents the temperature fields on the channel scale for four instants of time. One can clearly see vortices caused by the galloping detonation. Thus, it has been established that there exist critical values of the Mach number of the incident flow on which the qualitative and quantitative pattern of the flow depends. In the case where the combustible mixture inflows through the entire cross section, two different detonation regimes have been obtained: one with a stationary wave on the ledge and the other with a wave propagating towards the inlet cross section of the channel. In the case with a layer of the combustible mixture, depending on the Mach number of the incident flow, three detonation regimes are realized: one with a stationary wave on the ledge and the other two with a wave propagating towards the inlet cross section of the channel either in the form of a stationary-wave complex or in the regime of galloping layered detonation. With increasing Mach number the velocity of propagation of the wave towards the inlet cross section decreases, and at $M = M_{***}$ the wave goes to the steady-state regime.

If the ledge blocks the entire channel cross section, then the same regimes of detonation propagation take place, except for the steady-state one. The investigations performed make it possible to draw the conclusion that galloping layered detonation is realized only owing to the boundedness of the channel in the transverse direction. It is due to the formation of a complex wave structure of the flow, characteristic of which is the penetration of the shock wave formed in the inert gas layer into the combustible mixture layer before the detonation wave, as a result of which it gets heated and ignites. The process in general has a periodic character different from the usual cellular detonation in a homogeneous medium.

Initiation of Detonation in Rotating Channels. Traditionally, to initiate detonation, two approaches are used. The first approach is connected with firing the mixture by a weak energy source and subsequent transformation of combustion to detonation with the help of special combustion-intensifying devices. Thus the transition of normal combustion to detonation is realized. In the second approach, called direct initiation of detonation, it is formed due to the shock wave from some external energy source, e.g., explosion of an explosive, electrical or laser-induced breakdown. The role of the detonation initiator can be played by the shock wave ahead of a body flying with a supersonic velocity in a combustible mixture or a fixed body with a supersonic flow passing around it. A little more than ten years ago, we investigated in the one-dimensional approximation a new method of initiation using the effect of cumulation

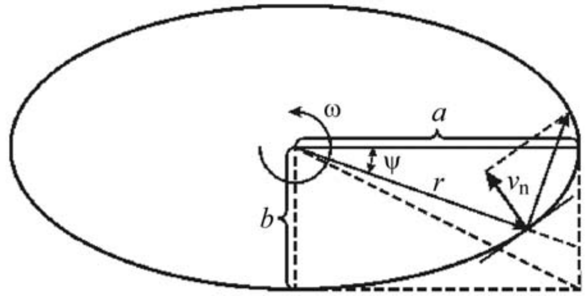


Fig. 24. Scheme illustrating the formulation of the problem.

of the initially weak shock wave near the axis or the center of symmetry [30–33]. A development of this approach for two-dimensional plane and axisymmetric flows was an original method of profiled tubes where a rather weak shock wave propagates in the channel and, due to its interaction with the wall and cumulation near the axis, favorable conditions for ignition and transformation of the compression shock to a detonation wave are formed [53, 54]. According to the hypersonic analogy [55, 56], the flow in the tube is similar to the axisymmetric, one-dimensional, nonstationary flow initiated by a piston whose radius varies with time following the tube profile. The results of investigation of the process of initiation of detonation made it possible to draw a conclusion about its efficiency in terms of minimization of both the energy consumption and the time of its formation [20]. Below we present the results of the investigation of problems formulated with the aim of developing and substantiating new methods of initiation of detonation.

Flow in a rotating elliptic cylinder. Consider an elliptic cylinder with base semiaxis lengths a and b shown in Fig. 24. The cylinder is filled with a stoichiometric propane–air mixture with pressure p_0, density ρ_0, and temperature T_0.

Let at the initial instant of time $t = 0$ the cylinder begin instantaneously to rotate with a constant angular velocity ω about the axis passing through the center of the base parallel to the element. As a result of the rotation, in the combustible mixture a two-dimensional flow in the base plane is formed. Hereafter, instead of the word combination elliptic cylinder the word ellipse is used. In so doing, near the parts of the cylinder wall moving towards the mixture, shock waves arise, and near the parts moving in the opposite direction rarefaction waves arise. Apparently, fairly powerful shock waves can cause detonation. The intensities of the shock waves and detonation waves arising by ellipse rotation will depend on the position of the point at its boundary. At the ends of the minor and major axes of the ellipse, the wave intensity is equal to zero by virtue of the equality to zero of the normal velocity component of the boundary at these points. They split the ellipse boundary into parts, on each of which either a shock wave or a rarefaction wave is formed. There also exist two points at the ellipse boundary symmetric about its center at which the normal velocity component of the boundary reaches its maximum. At these points the intensity of the shock waves arising at the initial instant of time is maximal. The position of the maximum points can be found in the general case for a cylinder with a base of any form. In the most compact form the condition has the form $\frac{d^2r^2}{ds^2} = 0$, where r is the distance to the center and s is the arc length. In polar coordinates (r, φ), the positions of the maximum points can be found from the equivalent condition $r^3\frac{d^2r}{d\varphi^2} + \left(\frac{dr}{d\varphi}\right)^4 = 0$. In the case of an ellipse, the maximum of the normal velocity component of the boundary is equal to $\omega(a-b)$ and is attained at the ends of the radius vectors located at an angle $\varphi = \arctan\,[(b/a)^{3/2}]$ to the major semiaxis counted off in the direction counter to the rotation. The above points correspond to the best conditions for fast initiation of detonation at large rotational velocities. However, at relatively low values of ω detonation can be initiated not immediately but after some time as a result of the complex interaction of the shock waves with the ellipse wall and with each other.

The investigations performed have fully confirmed the above scenarios of development of the process. Below we present the results of the calculations of the flow for the following values of the diagnostic variables: $a = 0.2$ m, $b = 0.1$ m, $p_0 = 1$ atm, and $T_0 = 20^{\circ}$C at various values of the angular velocity ω. At angular velocities higher than a certain critical velocity $\omega_{**} = 13{,}000$ rad/s, detonation is initiated immediately near two parts of the ellipse boundary symmetric about its center. At $\omega \to \infty$ these parts expand, tending to two quarters of the ellipse. At $\omega = \omega_{**}$ they

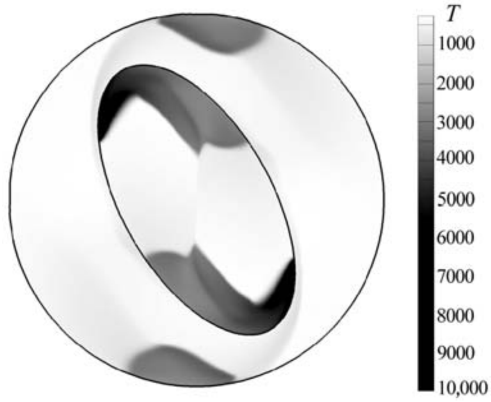


Fig. 25. Temperature fields under detonation of the mixture inside and outside a rotating elliptic cylinder confined in a circular cylinder.

degenerate to points. The position of these points obtained in the calculations is in good agreement with the analytical conclusions about the maximum of the normal velocity component. At angular velocities lower than a certain critical velocity $\omega_* = 6000$ rad/s, detonation inside the rotating cylinder is not initiated at all. In the range of velocities between ω_* and ω_{**}, detonation is formed not immediately but after some time as a result of the complex interaction of the shock waves. In so doing, the portions of the shock waves arising at the initial instant of time become with time more and more intense, and upon reaching the threshold value they transform to detonation waves. In the given case, the most favorable conditions for detonation are near the ends of the major axis of the ellipse, since at these points the curvature and the linear velocity are maximal. Note that even at a minimal angular rotational velocity ω_* leading to detonation, initiation occurs fairly fast — in a quarter of a cylinder rotation.

Interestingly, the considered two-dimensional problem has the following practically important three-dimensional analogy. Consider a channel having a special helical form obtained by simultaneous uniform rotation of an ellipse and its movement with a constant velocity along the channel axis. If a combustible mixture flows into such a channel along the axis with a given supersonic velocity, then, as a result of the flow–wall interaction a complex three-dimensional flow pattern will be formed and conditions for the initiation of detonation can be created. If the pitch of the helix is much larger than the ellipse sizes, then on the basis of the flat cross-section hypothesis [55, 56] we can use the result of the above-considered problem in the two-dimensional formulation for an approximate description of the three-dimensional processes at supersonic velocities of the inflowing mixture. The calculated data obtained at a certain value of ω can be applied to a whole spectrum of channels with different pitches of the helix $H = 2\pi U/\omega$.

Initiation of detonation outside the elliptic cylinder. Let a stationary propane–air mixture with the same temperature and pressure as in the previous problem be outside the elliptic cylinder. According to the flat cross-section hypothesis, the flow beyond a rotating elliptic cylinder can be interpreted as a supersonic flow past a helical body. Let us consider flows at various values of the angular rotational velocity of the ellipse ω. As would be expected, at an angular velocity exceeding the previously obtained critical velocity ω_{**} instantaneous initiation of detonation occurs. At lower velocities detonation is not formed. From the results obtained above for the flow inside the ellipse it follows that initiation of detonation at $\omega < \omega_{**}$ occurs some time after the onset of rotation as a result of the interaction of the shock waves with the ellipse walls and with each other. The dominant role is played here by the concavity of the ellipse wall. This suggests that to initiate detonation in the external flow problem, one has to prevent the propagation of diverging shock waves by bounding the flow region. For such boundedness, consider a circular cylinder with the axis coinciding with the axis of the elliptic cylinder. As the calculations have shown, at fixed $\omega < \omega_{**}$ and various values of the radius r of the circular cylinder, detonation in the combustible mixture is initiated at values of the circular cylinder r smaller than some critical r_*. With decreasing difference between r and the major semi-axis a of the ellipse, i.e., with decreasing gap between the cylinders, the value of the critical angular velocity ω_* necessary for detonation to be initiated decreases. At small gaps the formation of detonation is favored by the choking effect of the combustible mixture flow in the gap relative to the reference system associated with the elliptic cylinder.

Figure 25 shows the temperature field outside and inside (for comparison) the elliptic cylinder at $\omega = 12{,}000$ rad/s, $a = 0.2$ m, $b = 0.1$ m, and $r = 0.25$ m. The flows inside and outside the cylinder are independent, and the

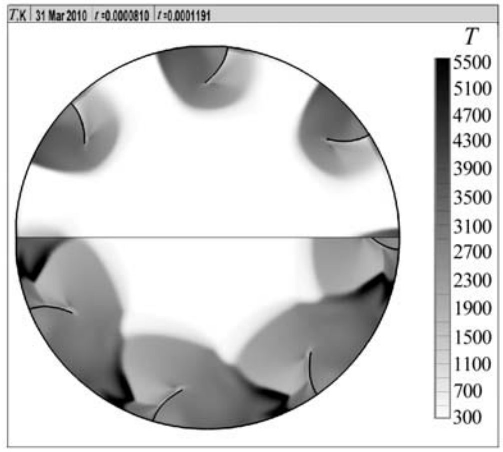


Fig. 26. Temperature field in a rotating circular cylinder with parabolic ledges uniformly distributed along its outer boundary.

simultaneous calculation therewith makes it possible to compare the angular velocities necessary for detonation to be initiated and the flow patterns on the whole.

Initiation of detonation by rotation of nonsmooth cylinders. The idea of considering cylindrical surfaces with fractures or asperities of various kinds is connected with the exploration of the possibility of rapid formation of detonation due to the cumulation effects at the surface fracture points. Below we present the results for two problems with the example of which the features of the flow and the formation mechanism of detonation are demonstrated visually.

Let, in a circular cylinder along its internal boundary, obstacles in the form of segments of parabolas be distributed uniformly. The parabola is constructed in the Cartesian coordinate system with origin at a point on a circle with the x axis directed along the external normal, and the y axis — along the tangent to the circle towards rotation. In so doing, the parabola segment emanates from the origin of coordinates and ends at the point ($x_0 = -0.04$ m, $y_0 = 0.02$ m). Consider flows at various rotational velocities. Below the results obtained for a cylinder of radius $r = 0.2$ m at the initial pressure and the temperature given previously are presented.

Figure 26 shows the temperature field at a rotational velocity $\omega = 7000$ rad/s. The upper half of the disk corresponds to the instant of time $t = 81$ μs, and the lower half — to 119 μs. In this case, detonation is formed just ahead of the obstacle in the vicinity of the point of its intersection with the cylinder. At lower rotational velocities no rapid initiation has been observed. Thus, $\omega = 7000$ rad/s is the critical velocity ω_{**}. According to the calculations, it decreases with increasing r according to the equality $v_{**} = \omega_{**}r = 1400$ m/s, where v_{**} is the linear rotational velocity of the cylinder boundary. Note that upon the start of motion at the intersection point of the circle with the parabola segment a shock wave moving at a velocity of 1640 m/s arises. The velocity of the ideal Chapman–Jouguet wave under the considered conditions is 1880 m/s. This points to the fact that in this case the initiation mechanism of detonation is similar to the soft mechanism of initiation of detonation by a piston leading to Chapman–Jouguet detonation. In the course of time, the detonation wave propagates along the cylinder surface, intensifying due to the centrifugal forces. Gradually a flow with a detonation wave converging to the center is formed in the form of a curvilinear heptagon (according to the number of obstacles) behind which a strongly nonuniform flow is observed.

An entirely different flow pattern is observed in the case of rotation of a star-shaped figure with rays in the form of segments of parabolas emanating from the center. This figure is composed of seven segments of parabolas obtained by the transformation of rotation by an angle $2\pi/7$ from the parabola segment given in the Cartesian coordinate system with origin at the center of rotation according to the formula $\gamma = 0.2\frac{x}{0.16}\left(1.1 - \frac{x}{0.16}\right)$. The required part of the parabola is isolated by points (0, 0) and (0.16 m, 0.2 m). Figure 27 shows the temperature field at a velocity $\omega = 7000$ rad/s. The upper half-disk corresponds to the instant of time $t = 245$ μs, and the lower one corresponds to $t = 350$ μs. In the given case, detonation is formed not at once but after rather a long time in which a quarter of a rotation is completed. In the process of rotation, shock waves diverging from the center are generated and interact with the parabolic waves and with each other. Detonation is originally initiated at the ends of the rays where the linear velocity is

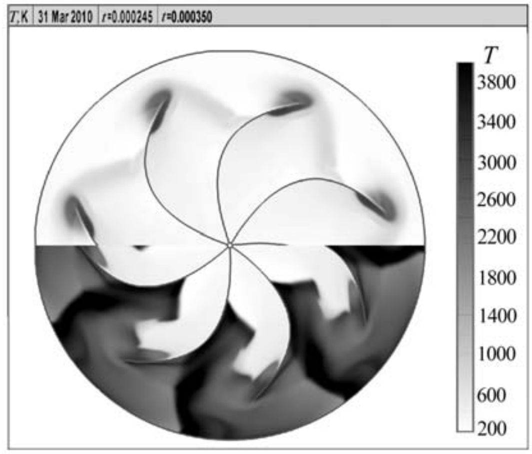


Fig. 27. Temperature field in a circular cylinder inside of which a star-like figure is rotating.

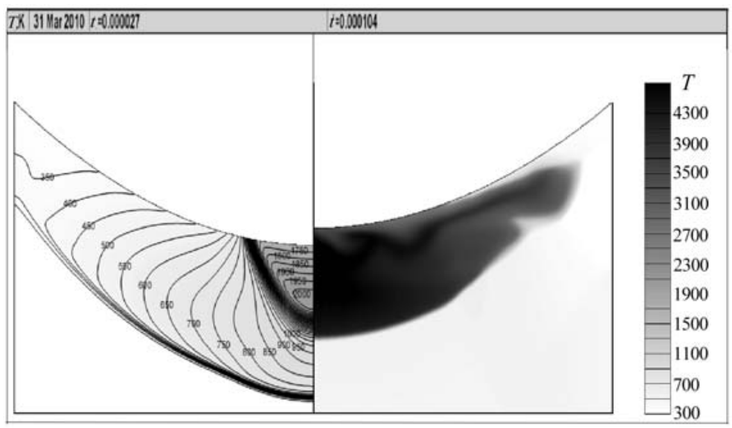


Fig. 28. Temperature field in a region with a parabolic wall experiencing harmonic oscillations.

maximum. Then the detonation wave diverges from the place of generation. It propagates as a narrow layer along the ray to the center and moves as an intensively diverging front to the cylinder boundary. The powerful shock wave formed upon the interaction of the detonation wave with the cylinder transforms to a convergent detonation wave with a broken front initiating reactions of the unburnt mixture in the space between the rays (lower half-disk in Fig. 27).

Flows in a Flat Chamber with Deformable Walls. Below we present the results of investigation of the process of detonation formation in regions of simple configuration bounded by deformable walls. It is assumed that due to deformation the volume can change. The considered two-dimensional problems, as the previous ones, can be used for an approximate description of three-dimensional high-speed flows in channels with a length-varying cross section.

The figures below show the temperature fields when the upper wall deforms as a sinusoid with given frequency and amplitude, retaining the parabolic form, and the other three walls are rectilinear and fixed. The influence of the oscillation frequency on the process of initiation of detonation is considered. The height of the side walls is 0.05 m, the width is 0.1 m, and the maximum amplitude in the middle of the deformable wall is 0.02 m.

Figure 28 illustrates the case with an oscillation period of 90 μs. To the left-hand half of the image corresponds the instant of time t = 27 μs and to the right-hand half — t = 104 μs. In the given case, the shock wave caused by the motion of the wall creates a nucleation site for combustion near the top of the parabolic wall, but detonation is not formed. Initiation of detonation occurs at the beginning of the second period of oscillations. According to the calculations, the critical period of oscillations for rapid initiation of detonation is T_{**} = 85 μs. At periods T larger than T_* = 300 μs detonation is absent. If T is in the interval (T_*, T_{**}), then detonation is initiated on any of the walls.

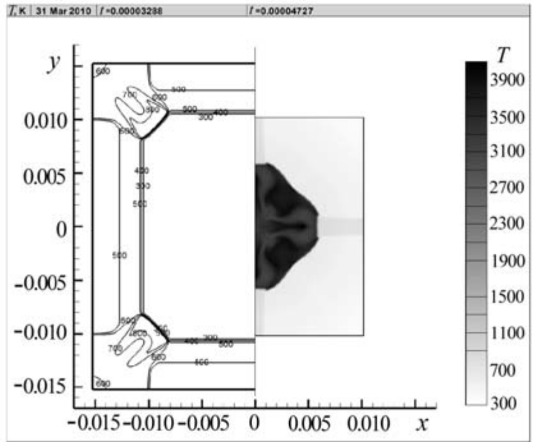


Fig. 29. Temperature field in a square region with a side length varying as a sinusoid.

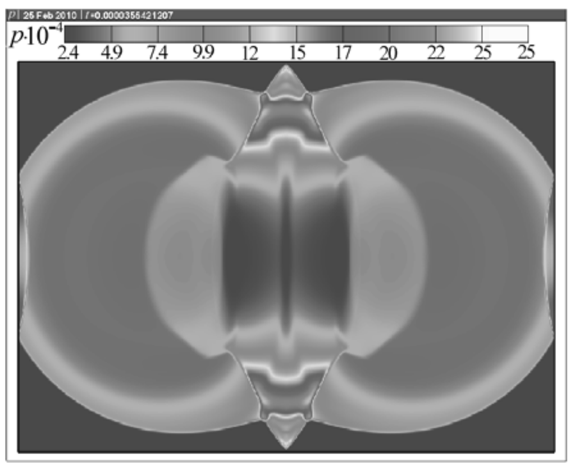
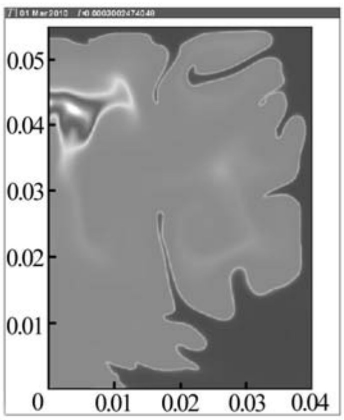


Fig. 30. Pressure field upon reflection of the wave from the symmetry axis.

The problem of detonation formation in a square region with a side length varying as a sinusoid has been investigated. Critical values of the oscillation periods have been obtained. Figure 29 shows the temperature field for two instants of time t = 33 and 47 μs at T = 100 μs when the oscillation amplitude of the square side length with respect to the value 0.04 m is 0.02 m. In the given case, the critical values of the oscillation period are equal: T_{**} = 85 μs; T_* = 85 μs. An analogous investigation of the initiation of detonation in a circle with a radius varying as a sinusoid with an oscillation amplitude 0.01 with respect to the value 0.02 m has shown that the critical values of T_{**} and T_* are smaller than the values obtained for the square. For example, at T = 100 μs detonation in the circle is not formed.

Initiation of Detonation by a Ring Discharge. To clarify the question on the possibility of remote initiation of detonation, we performed computational modeling of a toroidal electric discharge in a methane–oxygen mixture. The torus radius is 0.025 m, and the radius of the circular cross section of the discharge is 0.005 m. It is assumed that at a discharge an energy equal to the discharge energy is supplied to the gas instantaneously or uniformly in time during the given period in the toroidal zone. The discharge is in the open space or in the middle of a cylindrical tube with closed ends of radius 0.055 m and length 0.08 or 0.32 m in the plane perpendicular to the tube axis. From the formulation of the problem, axial symmetry of the flow initiated by the discharge follows.

According to the calculations, if the discharge energy exceeds 350 J, then direct initiation of detonation occurs. Otherwise the originally initiated overcompressed detonation wave transforms to a shock wave followed by a slow combustion zone. In all cases, upon reflection of the initial toroidal shock from the symmetry axis, a flow with a fast jet near the symmetry axis is formed, and ahead of this jet, as ahead of a body moving with a high velocity, an attached conical shock wave propagates. The wave fronts separating the flow region from the stationary combusti-



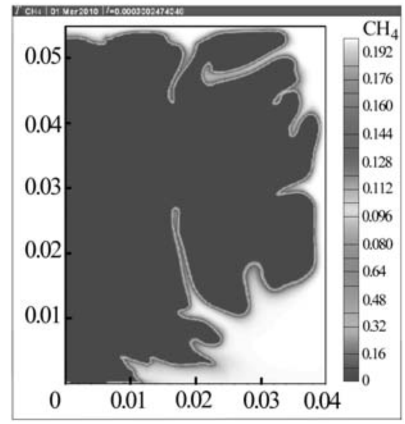


Fig. 31. Temperature field at the final stage of combustion.

Fig. 32. Mass fraction of methane at the final stage of combustion.

ble mixture form a complex consisting of an incident toroidal wave and a Mach disk "pierced" by the conical wave (Fig. 30).

As the conical wave propagates, it attenuates and grows blunt, which is due to the decrease in the axial jet velocity. However, the formation process of axial jets and conical shock waves is repeated in the case of a bounded space of the tube due to the arrival of new shock waves reflected from its inner surface.

Interestingly, in the case of energy release by a discharge in the course of 15 μs, the time of arrival of the wave at the symmetry axis increases to 25 μs compared to 16 μs by an instantaneous discharge.

To obtain information on the flow on a large time interval, calculations in a short tube with a low discharge energy of 30 J were made. In the given case, the flow pattern is like that described above up to the instant of time at which the wave front formed by a toroidal wave, a Mach ring, and a conical wave begins to interact with the end walls of the tube. At the following instants of time, there appears a complex picture of chaotic interaction of the shock waves with each other, with the tube walls, with the symmetry axis, and with the energy-release zone in which high-temperature combustion products of the methane–oxygen mixture are situated. The combustion zone expands, deforms, and turbulizes (Fig. 31).

Figure 32 presents the field of the mass fraction of methane. It is well seen that the hot zone boundaries coincide with the boundary of unburnt methane. The observed phenomenon is due to the development of Richtmeier–Meshkov instability under the action of the systems of shock waves that are due to the axial symmetry of the combustion chamber and its boundedness in the radial and axial directions. The ignition front, being a contact discontinuity separating gases with various widely differing densities, strongly deforms and the combustion propagation becomes turbulent.

CONCLUSIONS

Analytical and numerical investigation of flows with detonation waves have been carried out from the point of view of the model of an infinitely thin detonation wave and with account for the single-stage combustion kinetics. New results for two-dimensional cellular detonation have been obtained. A new regime of galloping detonation has been revealed.

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NOTATION

a_0, velocity of sound in the incident flow; a, b, lengths of ellipse semiaxes; c_1, c_2, incident flow parameter in a cross section with area S_0; D, detonation-wave velocity; D_1, velocity of the Chapman–Jouguet wave; e, total energy of the medium; $f(x)$, function describing the form of the lateral wall of the channel; H, total enthalpy of the mixture; I, driving force in the axial direction; I_S, mean momentum; J, mean specific momentum; L, channel length; M, Mach number; N, number of mixture components; p, pressure of the combustible mixture; Q, specific heat release; R, radius of the flat closed end of the channel; R_0, universal gas constant; R_g, gas constant; r, distance to the symmetry axis of the channel; S, cross section area of the channel; T, temperature of the combustible mixture; t, time; U, incident-wave velocity; u, v, longitudinal and radial velocity components of the gas; x, symmetry axis of the channel; α, half-space angle of the channel; β, mass fraction of the fuel; γ, Poisson adiabatic index; ε, specific internal energy; ρ, density of the combustible mixture; τ, chemical reaction time. Subscripts: n, normal; t, tangential; w, wall; i, mixture-component number; g, gas.

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